

Engineering Economic Analysis

FOURTEENTH EDITION

Chapter 3

Interest & Equivalence

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Chapter Outline

- Computing Cash Flows
- Time Value of Money
- Equivalence
- Single Payment Compound Interest Formulas
- Nominal & Effective Interest Rates

Learning Objectives

- Understand time value of money
- Distinguish between simple & compound interest
- Understand cash flow equivalence
- Solve problems using single payment compound interest formulas
- Solve problems using spreadsheet factors

Vignette: A Prescription for Success

- Complex tablet press operation
- Significant scrap & tablet press downtime
- Equipment modification to 3 presses cost \$90,000
- Impact of modifications:
 - Each batch finished in 16 hrs (\Leftrightarrow 24 hrs)
 - Product yield increased to 96.6% (\Leftrightarrow 92.4%)
 - Production was reduced to 2 shifts (\Leftrightarrow 3)
 - 240 batches processed in one year
 - First year savings of \$10 million



Vignette: A Prescription for Success

- Product value = \$240 M /yr; what is value of one batch?
- How many batches for breakeven on initial \$27 K investment? (assume 4.2% yield improvement)
- What is project's present value?
 - Assume interest rate is 15%,
 - Savings are a single end-of-year cash flow, &
 - \$90,000 investment is at time 0.
- If 1 batch produced per day, how often are savings actually compounded?

Computing Cash Flows

- Would you rather
 - Receive \$1000 today; or
 - Receive \$1000 10 years from today?
- Answer: Today!
- Why?
 - I could invest \$1000 today to make more money
 - I could buy a lot of stuff today with \$1000
 - Who knows what will happen in 10 years

Computing Cash Flows

- Cash flows are
 - Costs (disbursements) = a negative number
 - Benefits (receipts) = a positive number
- Because money is more valuable today than in the future, we need to describe cash receipts & disbursements at time they occur.

Example 3-1

Cash flows of 2 payment options

EXAMPLE 3-1

A machine will cost \$30,000 to purchase. Annual operating and maintenance costs (O&M) will be \$2000. The machine will save \$10,000 per year in labor costs. The salvage value of the machine after 5 years will be \$7000. Draw the cash flow diagram.

Example 3-1

Cash flows of 2 payment options

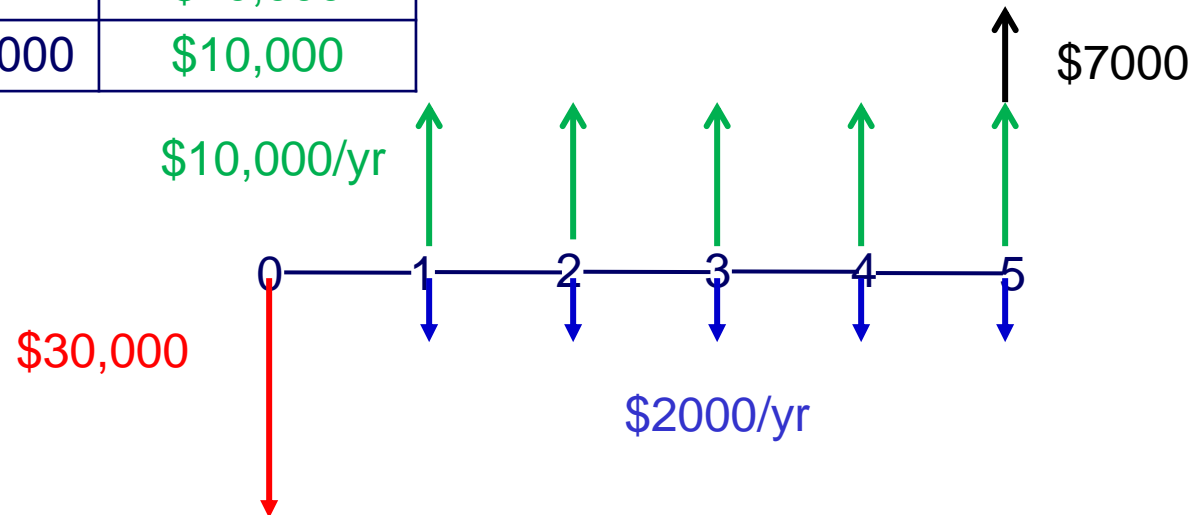
Purchase a new \$30,000 machine,

- O&M costs = \$2000/yr
- Savings = \$10,000/yr
- Salvage value at Yr 5 = \$7000

Draw the cash flow diagram

Example 3-1, Cash flows

End of Year	Costs & SV	Savings
0 (now)	-\$30,000	\$10,000
1	-2000	\$10,000
2	-2000	\$10,000
3	-2000	\$10,000
4	-2000	\$10,000
5	-2000+7000	\$10,000



Example 3-2

Cash flow for repayment of a loan

EXAMPLE 3-2

A man borrowed \$1000 from a bank at 8% interest. He agreed to repay the loan in two end-of-year payments. At the end of each year, he will repay half of the \$1000 principal amount plus the interest that is due. Compute the borrower's cash flow.

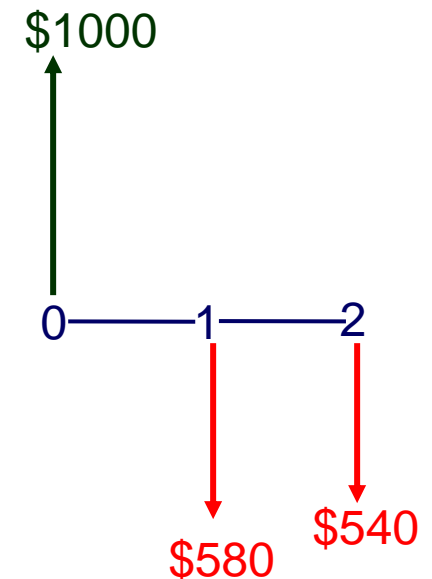
Example 3-2

Cash flow for repayment of a loan

To repay a loan of \$1000 at 8% interest in 2 years

- Repay half of \$1000 plus interest at the end of each year

Yr	Interest	Balance	Repayment	Cash Flow
0		1000		1000
1	80	500	500	-580
2	40	0	500	-540



Time Value of Money

Money has value

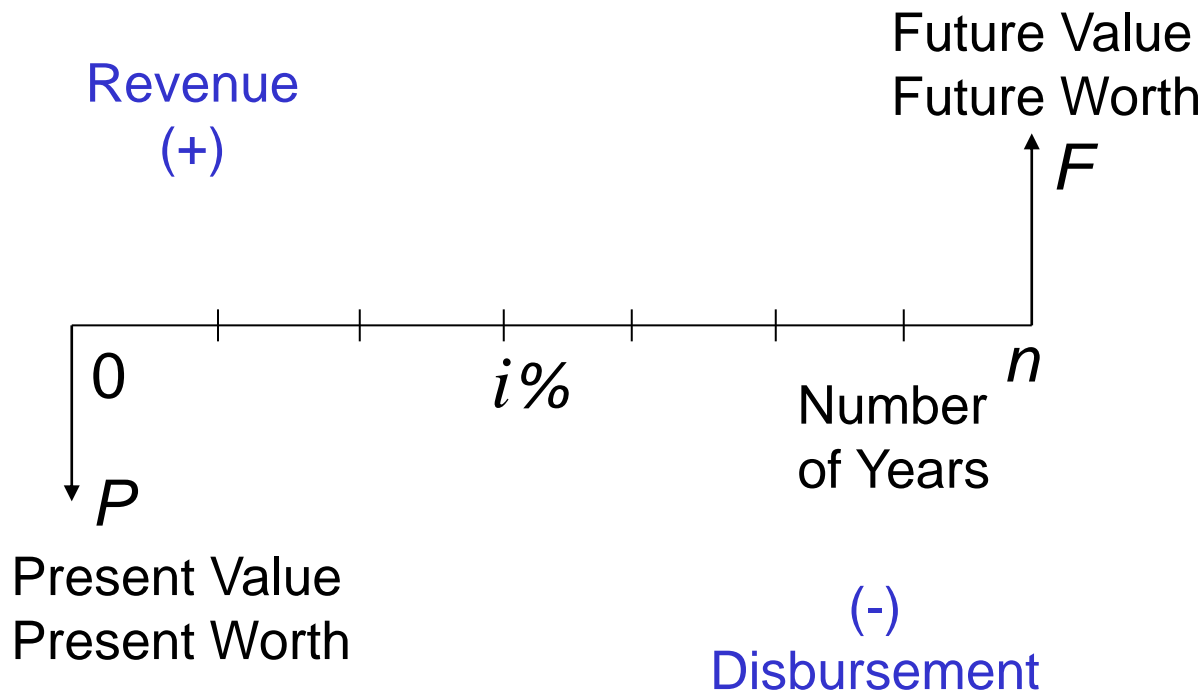
- Money can be leased or rented
- Payment is called interest
- If you put \$1000 in a bank at 4% interest for one time period you will receive back your original \$1000 plus \$40

Original amount to be returned = \$1000

Interest to be returned = $\$1000 \times .04 = \40

Cash Flow Diagram

Invest P dollars at $i\%$ interest & receive F dollars after n years



Simple Interest on Loan

Is computed only on original sum—does not include interest earned or owed

P borrowed for n years

Total interest owed = $P \times i \times n$

- P = present sum of money
- i = interest rate
- n = number of periods (years)

Simple interest = \$1000 x .04/period x 2 periods = \$80

Example 3-3

Simple Interest Calculation

EXAMPLE 3-3

You have agreed to loan a friend \$5000 for 5 years at a simple interest rate of 8% per year. How much interest will you receive from the loan? How much will your friend pay you at the end of 5 years?

Example 3-3

Simple Interest Calculation

Loan of \$5000 for 5 yrs at simple interest rate of 8%

Total interest owed = $\$5000(8\%)(5) = \2000

Amount due at end of loan = $\$5000 + 2000 = \7000

Compound Interest

- Interest computed on unpaid balance,
 - includes the principal
 - any unpaid interest from the preceding period

$$P = \frac{F}{(1 + i)^n}$$

$$F = P(1 + i)^n$$

Compound Interest on Loan

- Compound interest is computed on unpaid debt & unpaid interest
- Total interest earned = $P(1 + i)^n - P$
 - Where
 - P = present sum of money
 - i = interest rate
 - n = number of periods (years)

$$\text{Interest} = \$1000 \times (1+.04)^2 - \$1000 = \$81.60$$

Compound Interest

For compound interest

$$F_1 = 5000(1 + 0.04)^1 = \$5200$$

$$F_2 = 5200(1 + 0.04)^1 = \$5408$$

$$F_3 = 5408(1 + 0.04)^1 = \$5624$$

Differences from simple interest magnify as # of periods & interest rates increase

Compound Interest

For compound interest

$$F_1 = P(1 + i)$$

$$F_2 = F_1(1 + i) = P(1 + i)(1 + i) = P(1 + i)^2$$

$$F_3 = F_2(1 + i) = P(1 + i)^2(1 + i) = P(1 + i)^3$$

After n periods

$$F = P(1 + i)^n$$

Which is true?

- A. $P = F(1 + i)^n$
- B. $P = F(1 + n)^i$
- C. $P = F/(1 + i)^n$
- D. $P = F/(1 + n)^i$
- E. I don't know

Which is true?

- A. $P = F(1 + i)^n$
- B. $P = F(1 + n)^i$
- C. $P = F / (1 + i)^n$
- D. $P = F / (1 + n)^i$
- E. I don't know

Example 3-4

Compound Interest Calculation

EXAMPLE 3-4

To highlight the difference between simple and compound interest, rework [Example 3-3](#) using an interest rate of 8% per year compound interest. How will this change affect the amount that your friend pays you at the end of 5 years?

Original loan amount (original principal) = \$5000

Loan term = 5 years

Interest rate charged = 8% per year compound interest

Example 3-4

Compound Interest Calculation

Loan of \$5000 for 5 yrs at 8%

Year	Balance at the Beginning of the year	Interest	Balance at the end of the year
1	\$5,000.00	\$400.00	\$5,400.00
2	\$5,400.00	\$432.00	\$5,832.00
3	\$5,832.00	\$466.56	\$6,298.56
4	\$6,298.56	\$503.88	\$6,802.44
5	\$6,802.44	\$544.20	\$7,346.64

Repaying a Debt

Plan #1: Constant Principal

Repay of a loan of \$5000 in 5 yrs at interest rate of 8%
 Plan #1: Constant principal payment plus interest due

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$1,000.00	\$1,400.00
2	\$4,000.00	\$320.00	\$4,320.00	\$320.00	\$1,000.00	\$1,320.00
3	\$3,000.00	\$240.00	\$3,240.00	\$240.00	\$1,000.00	\$1,240.00
4	\$2,000.00	\$160.00	\$2,160.00	\$160.00	\$1,000.00	\$1,160.00
5	\$1,000.00	\$80.00	\$1,080.00	\$80.00	\$1,000.00	\$1,080.00
	Subtotal			\$1,200.00	\$5,000.00	\$6,200.00

Repaying a Debt

Plan #2: Interest Only

Repay of a loan of \$5000 in 5 yrs at interest rate of 8%

Plan #2: Annual interest payment & principal payment at end of 5 yrs

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
2	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
3	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
4	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
5	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$5,000.00	\$5,400.00
	Subtotal			\$2,000.00	\$5,000.00	\$7,000.00

Repaying a Debt

Plan #3: Constant Payment

Repay of a loan of \$5000 in 5 yrs at interest rate of 8%

Plan #3: Constant annual payments

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$852.28	\$1,252.28
2	\$4,147.72	\$331.82	\$4,479.54	\$331.82	\$920.46	\$1,252.28
3	\$3,227.25	\$258.18	\$3,485.43	\$258.18	\$994.10	\$1,252.28
4	\$2,233.15	\$178.65	\$2,411.80	\$178.65	\$1,073.63	\$1,252.28
5	\$1,159.52	\$92.76	\$1,252.28	\$92.76	\$1,159.52	\$1,252.28
	Subtotal			\$1,261.41	\$5,000.00	\$6,261.41

Repaying a Debt

Plan #4: All at Maturity

Repay of a loan of \$5000 in 5 yrs at interest rate of 8%

Plan #4: All payment at end of 5 years

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$0.00	\$0.00	\$0.00
2	\$5,400.00	\$432.00	\$5,832.00	\$0.00	\$0.00	\$0.00
3	\$5,832.00	\$466.56	\$6,298.56	\$0.00	\$0.00	\$0.00
4	\$6,298.56	\$503.88	\$6,802.44	\$0.00	\$0.00	\$0.00
5	\$6,802.44	\$544.20	\$7,346.64	\$2,346.64	\$5,000.00	\$7,346.64
	Subtotal			\$2,346.64	\$5,000.00	\$7,346.64

4 Repayment Plans

- Differences:
 - Repayment structure (repayment amounts at different times)
 - Total payment amount
- Similarities:
 - All interest charges were calculated at 8%
 - All repaid a \$5000 loan in 5 years

Equivalence

- If a firm believes 8% was reasonable, it would have no preference about whether it received \$5000 now or was paid by any of the 4 repayment plans.
- The 4 repayment plans are equivalent to one another & to \$5000 now at 8% interest

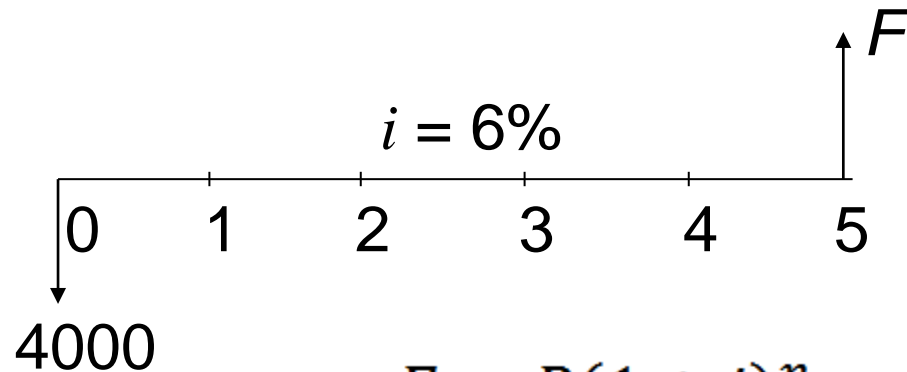
Use of Equivalence in Engineering Economic Studies

- Using **equivalence**, one can convert different types of cash flows at different points of time to an equivalent value at a common reference point
- Equivalence depends on interest rate

Example

If you were to receive \$4000 today to invest at 6% interest, what would this be equivalent to in 5 years?

Given: $P = 4000$, $i = 6\%$, $n = 5$



$$F = P(1 + i)^n$$

$$F = 4000(1 + 0.06)^5 = \$5352.90$$

You deposit \$100 in account earning 5%

After 4 years the value in account is

- A. -\$121.55
- B. \$121.55
- C. \$121.67
- D. \$431.01
- E. None of the above

$$F = P(1 + i)^n$$

You deposit \$100 in account earning 5%.

After 4 years the value in account is

- A. -\$121.55
- B. **\$121.55** $F = 100(1.05)^4$
- C. \$121.67
- D. \$431.01
- E. None of the above

Interest Formulas

Notation:

i = Interest rate per interest period

n = Number of interest periods

P = Present sum of money (Present worth)

F = Future sum of money (Future worth)

Basic factors

Equation:	$P = F / (1 + i)^n$
Factor:	$P = F(P/F, i, n)$
Function:	=PV(rate, nper, pmt, [FV], [type])
Equation:	$F = P(1 + i)^n$
Factor:	$F = P(F/P, i, n)$
Function:	=FV(rate, nper, pmt, [PV], [type])

Factors & Functions

<u>Variable</u>	<u>Engineering Economy</u>	<u>Spreadsheets</u>
Present value	P	PV
Future value	F	FV
Uniform series	A	PMT
Interest rate	i	RATE
Number of periods	n	NPER

Notation for Calculating a Future Value

Formula:

$F = P(1 + i)^n$ is the
single payment compound amount factor

Functional notation:

$$F = P(F/P, i, n) \quad F = 5000(F/P, 6\%, 10)$$

$F = \cancel{P}(F/\cancel{P})$ is dimensionally correct

In Excel,

`=FV(rate,nper,pmt,[pv],[type])`

Notation for Calculating a Present Value

$$P = F \left(\frac{1}{1+i} \right)^n = \frac{F}{(1+i)^n} \text{ is the}$$

Single payment present worth factor

Functional notation:

$$P = F(P/F, i, n) \quad P = 5000(P/F, 6\%, 10)$$

In Excel,

`=PV(rate,nper,pmt,[fv],[type])`

Excel financial functions

=PV(rate, nper, pmt, [fv], [type])

=FV(rate, nper, pmt, [pv],[type])

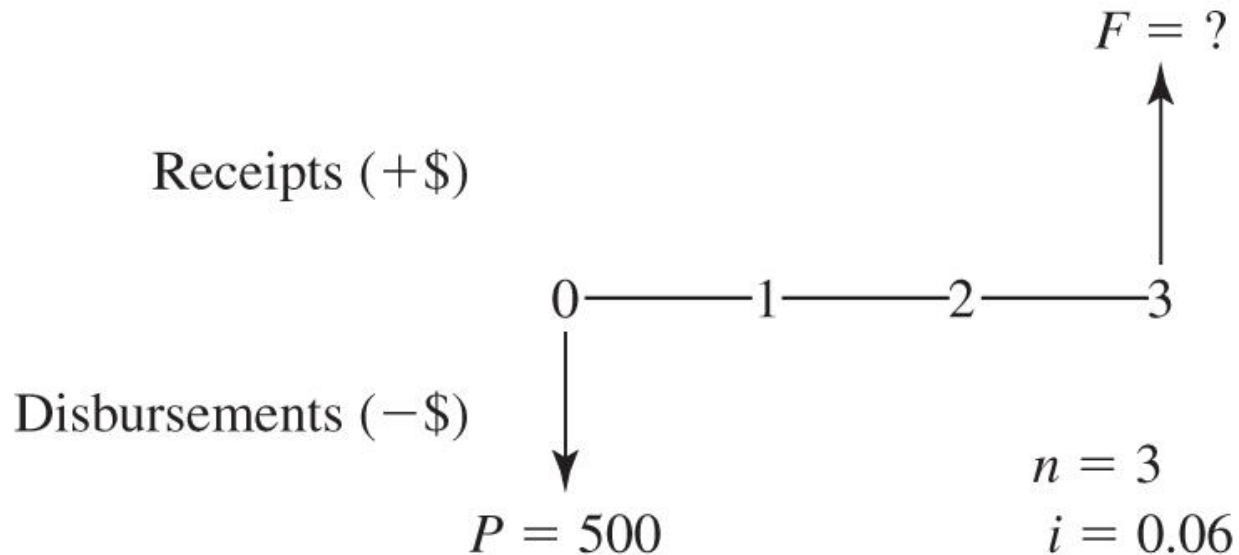
=PMT(rate, nper, pv,[fv],[type])

=NPER(rate, pmt, pv, [fv], [type])

=RATE(nper, pmt, pv, [fv], [type],[guess])

Example 3-5

\$500 is deposited today. What is it worth in 3 years at 6% interest?



Example 3-5

- $$F = P(1 + i)^n$$

$$= 500(1+.06)^3 = 500(1.191) = \$595.51$$
- $$F = P(F/P)$$

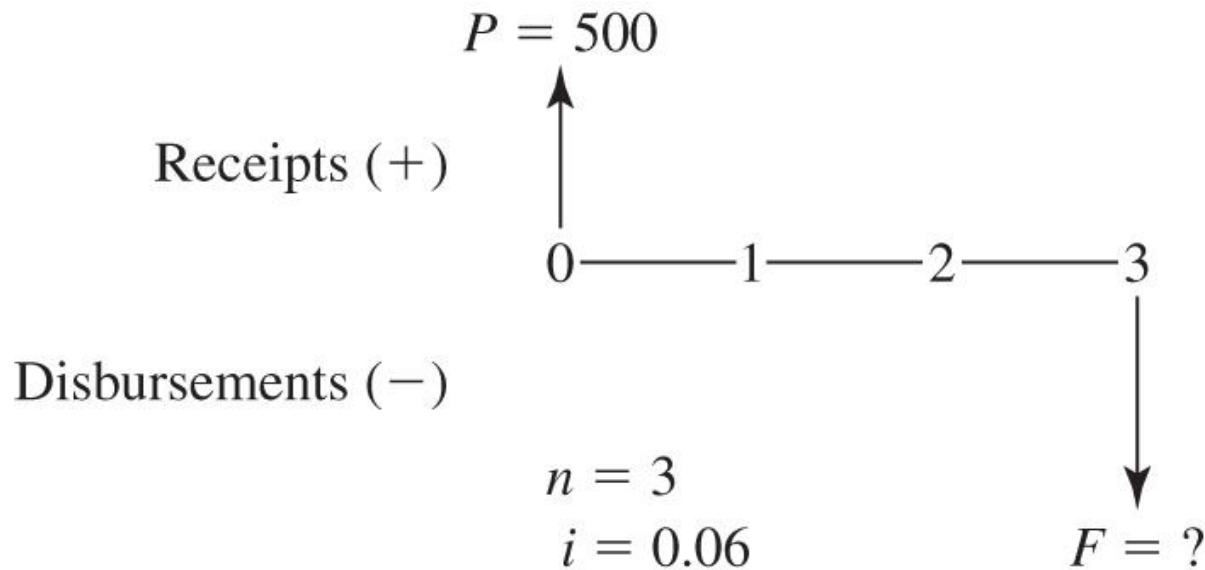
$$= 500(F/P, 6\%, 3) = 500(1.191) = \$595.50$$

	A	B	C	D	E	F	G	H
1	ID	<i>i</i>	<i>n</i>	<i>PMT</i>	<i>PV</i>	<i>FV</i>	Answer	Formula
2	3-5	6%	3	0	-500		\$595.51	=FV(B2,C2,D2,E2)

=FV(rate, nper, pmt, [pv], [type])

Example 3-5

From the bank's point of view, are the numbers different?
No—only the sign changes.



How Excel computes this

=FV(rate,nper,pmt,pv)

$i = 6\%$ $nper = 3$ $pmt = 0$ $pv = 500$

$FV = -595.508$

Excel uses the following equation:

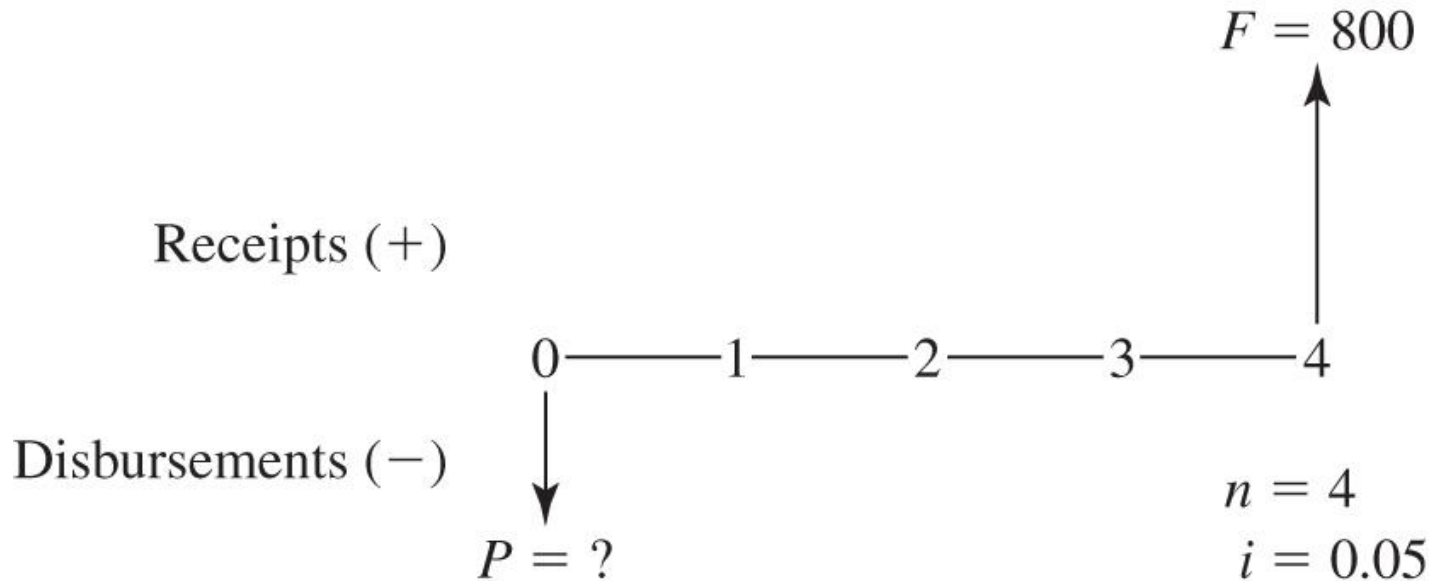
$$PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right] + FV(1 + i)^{-n} + PV = 0$$

So PMT, FV, & PV cannot be the same sign

Example 3-6

EXAMPLE 3-6

If you wish to have \$800 in a savings account at the end of 4 years, and 5% interest will be paid annually, how much should you put into the savings account now?



Example 3-6

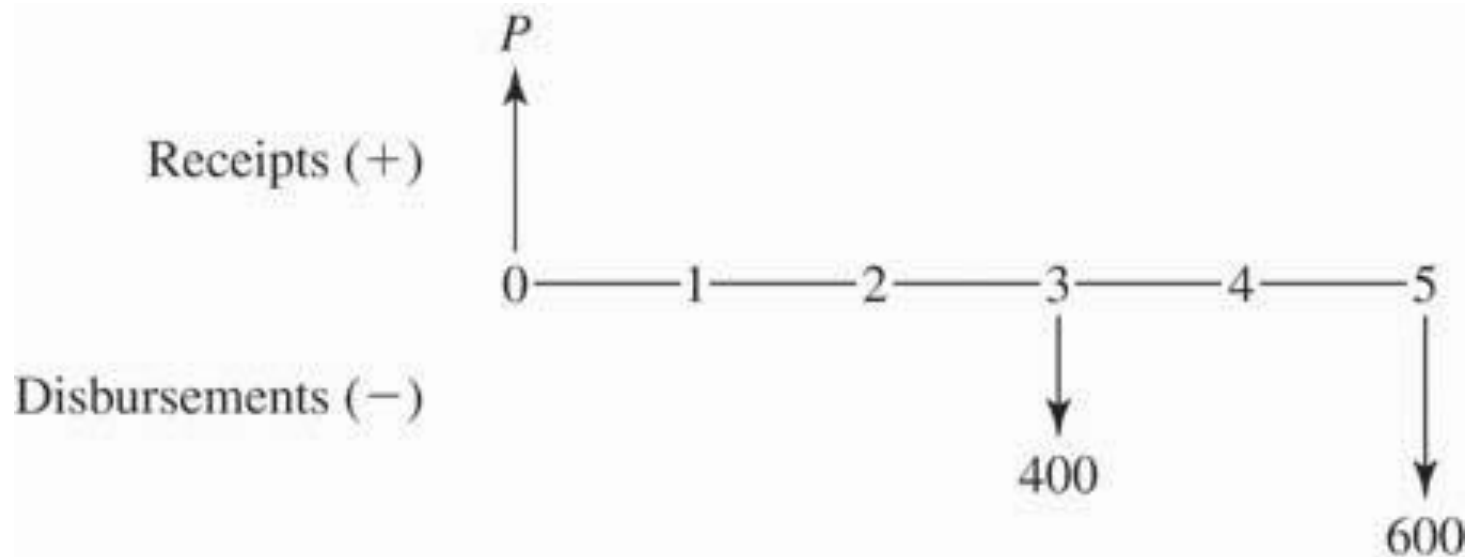
$$P = F / (1+i)^n = 800/(1+0.05)^{-4} = \$658.16$$

$$P = F(P/F, i, n) = 800(P/F, 5\%, 4) = 800(0.8227) = 658.16$$

or

	A	B	C	D	E	F	G	H
1	ID	<i>i</i>	<i>n</i>	<i>PMT</i>	<i>PV</i>	<i>FV</i>	Answer	Formula
2	3-6	5%	4	0		800	-\$658.16	=PV(B2,C2,D2,F2)
3								=PV(rate,nper,pmt,[fv],[type])

2 Cash Outflows



$$i = 12\%$$

2 Cash Outflows

$$P = 400(P/F, 12\%, 3) + 600(P/F, 12\%, 5)$$

$$= 400(0.7118) + 600(0.5674) = \$625.16$$

or

	A	B	C	D	E	F	G	H
1	ID	<i>i</i>	<i>n</i>	<i>PMT</i>	<i>PV</i>	<i>FV</i>	Answer	Formula
2	2 CF	12%	3	0		-400	\$284.71	=PV(B2,C2,D2,F2)
3		12%	5	0		-600	\$340.46	=PV(B3,C3,D3,F3)
4							\$625.17	=G2+G3

You deposit \$100 in an account earning 5%

After 4 years the value in the account is:

A. -121.55

B. 121.55

C. 121.66

D. -121.66

A. Something else or I don't know

You deposit \$100 in an account earning 5%

After 4 years the value in the account is:

A. -121.55

B. **121.55** $= 100(F/P, 5\%, 4) = 100(1.216)$
 $= FV(5\%, 4, 0, -100)$

C. 121.66

D. -121.66

E. Something else or I don't know

You need \$6000 in 3 years as a down payment on a car.

If your savings earn 0.25% interest per month, how much do you need to deposit today to have \$6000 in 3 years?

- A. 5955.22
- B. 5490.85
- C. 5484.20
- D. 2070.19
- E. I don't know

You need \$6000 in 3 years as a down payment on a car.

If your savings earn 0.25% interest per month, how much do you need to deposit today to have \$6000 in 3 years?

A. 5955.22

B. 5490.85

C. **5484.20**

$$= 6000(P/F, 0.25\%, 36) = 6000(0.9140)$$

$$= PV(0.25\%, 36, 0, -6000)$$

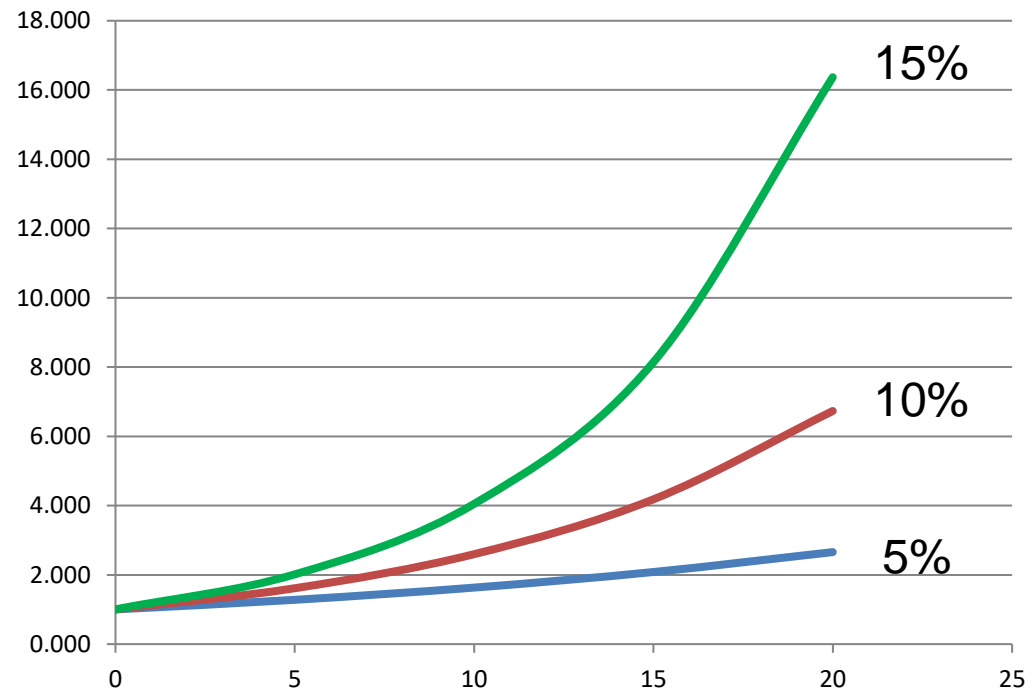
D. 2070.19

E. I don't know

Example 3-7 Single Payment Compound Interest Formulas

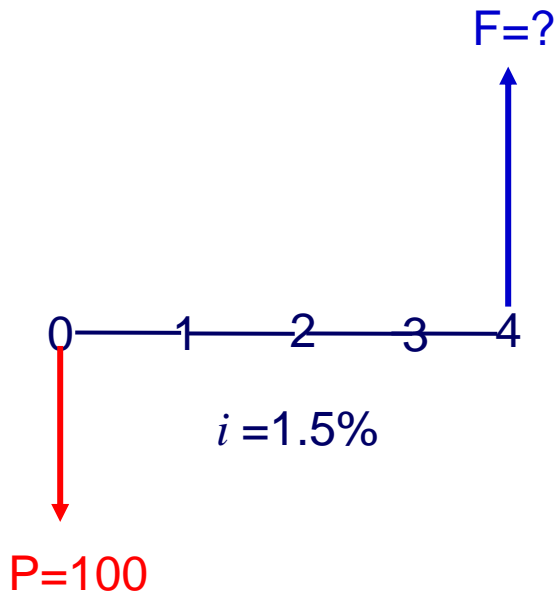
Tabulate the future value factor for interest rates of 5%, 10%, & 15% for n 's from 0 to 20 (in 5's).

n	5%	10%	15%
0	1.000	1.000	1.000
5	1.276	1.611	2.011
10	1.629	2.594	4.046
15	2.079	4.177	8.137
20	2.653	6.727	16.367



Example 3-8 Single Payment Compound Interest Formulas

\$100 were deposited in a saving account (pays 6% compounded quarterly) for 1 year



$$i_{\text{qtr}} = 1.5\%, n = 4 \text{ quarters}$$

$$F = P(1 + i)^n = 100(1 + 0.015)^4 = \$106.14$$

$$F = P(F/P, i, n) = 100(F/P, 1.5\%, 4) = 100(1.061) = \$106.10$$

	A	B	C	D	E	F	G	H
1	ID	i	n	PMT	PV	FV	Answer	Formula
2	3-8	1.5%	4	0	-100		\$106.14	=FV(B2,C2,D2,E2)

Nominal & Effective Interest

- Nominal interest rate/year: the annual interest rate w/o considering the effect of any compounding.
 - 12%/year
- Interest rate/period: the nominal interest rate/year divided by the number of interest compounding periods.
 - $(12\%/year)/(12 \text{ months/year}) = 1\%/month$

Effective Interest Rate

The *effective interest rate* is given by the formula:

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

where r = nominal annual interest rate

m = number of compounding periods per year

Example 3-9 Nominal & Effective Interest Rates

If a credit card charges 1.5% interest every month, what are the nominal & effective interest rates per year?

$$r = 12 \times 1.5\% = 18\%$$

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

$$i_a = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 0.1956 = 19.56\%$$

	A	B	C	D	E
1	Nominal annual rate, r	Periods per year, m	Effective rate, i_a	Answer	Spreadsheet Function
2	18.0%	12		19.56%	=EFFECT(A2,B2)

Example 3-10 Application of Nominal & Effective Interest Rates

"If I give you \$100 today, you will write me a check for \$120, which you will redeem or I will cash on your next payday in 2 weeks."

Bi-weekly interest rate = $(\$120 - 100)/100 = 20\%$

Nominal annual rate = $20\% * 26 = 520\%$

$$i_a = \left(1 + \frac{5.20}{26}\right)^{26} - 1 = 113.48 = 11,348\%$$

End of year balance owed = \$100 principal + \$11,348 interest

$$F = P(1 + i)^n = 100(1 + 0.20)^{26} = \$11,448$$

A credit card's APR is 12% with monthly compounding

What is the effective interest rate?

- A. 12.00%
- B. 14.4%
- C. 4.095%
- D. 12.68%
- E. None of the above

A credit card's APR is 12% with monthly compounding

What is the effective interest rate?

A. 12.00%

B. 14.4%

C. 4.095%

D. 12.68%

E. None of the above

$$= \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

Example 3-11 Application of Continuous Compounding

6% interest compounded continuously.

$$\begin{aligned}\text{Effective interest rate} &= e^r - 1 \\ &= e^{0.06} - 1 = 0.0618 \\ &= 6.18\%\end{aligned}$$