Engineering Economic Analysis

FOURTEENTH EDITION

Chapter 3

Interest & Equivalence

Donald G. Newnan
San Jose State University

Ted G. Eschenbach
University of Alaska Anchorage

Jerome P. Lavelle
North Carolina State University

Neal A. Lewis
Fairfield University

OXFORD
UNIVERSITY PRESS

Chapter Outline

- Computing Cash Flows
- Time Value of Money
- Equivalence
- Single Payment Compound Interest Formulas
- Nominal & Effective Interest Rates

Learning Objectives

- Understand time value of money
- Distinguish between simple & compound interest
- Understand cash flow equivalence
- Solve problems using single payment compound interest formulas
- Solve problems using spreadsheet factors

Vignette: A Prescription for Success

- Complex tablet press operation
- Significant scrap & tablet press downtime
- Equipment modification to 3 presses cost $90,000
- Impact of modifications:
  - Each batch finished in 16 hrs (⇔24 hrs)
  - Product yield increased to 96.6% (⇔92.4%)
  - Production was reduced to 2 shifts (⇔3)
  - 240 batches processed in one year
  - First year savings of $10 million

Vignette: A Prescription for Success

- Product value = $240 M /yr; what is value of one batch?
- How many batches for breakeven on initial $27 K investment? (assume 4.2% yield improvement)
- What is project's present value?
  - Assume interest rate is 15%,
  - Savings are a single end-of-year cash flow, &
  - $90,000 investment is at time 0.
- If 1 batch produced per day, how often are savings actually compounded?

Computing Cash Flows

- Would you rather
  - Receive $1000 today; or
  - Receive $1000 10 years from today?
- Answer: Today!
- Why?
  - I could invest $1000 today to make more money
  - I could buy a lot of stuff today with $1000
  - Who knows what will happen in 10 years

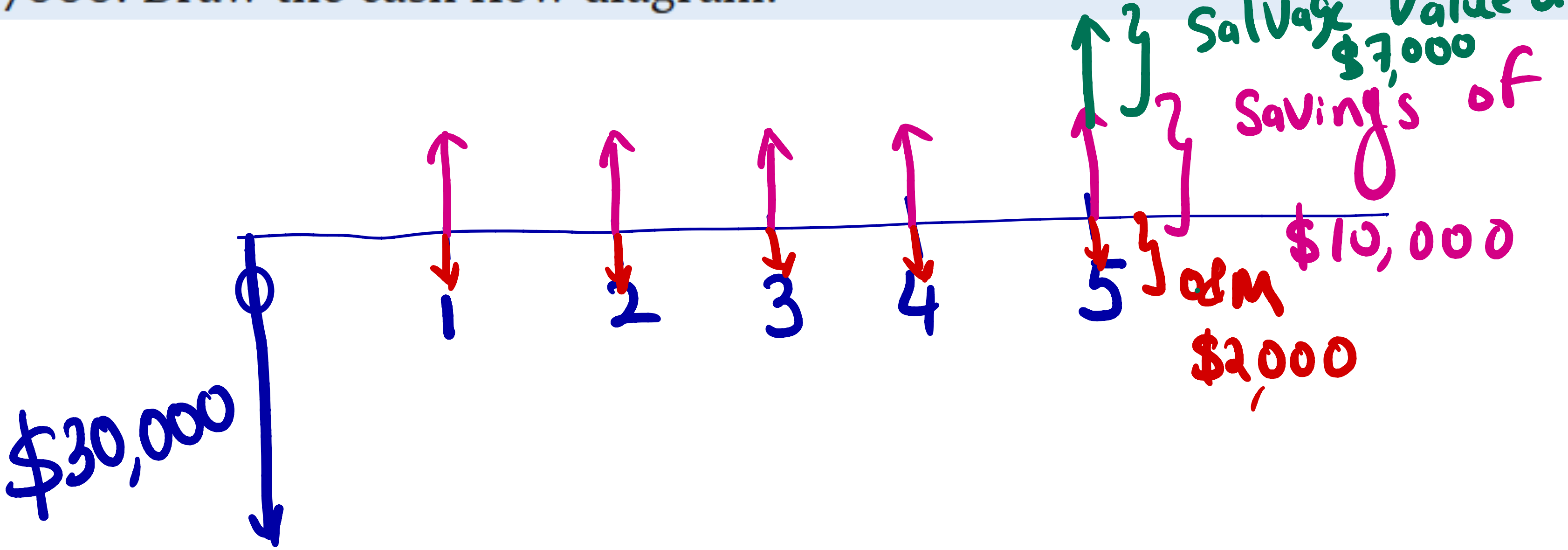
Computing Cash Flows

- Cash flows are
  - Costs (disbursements) = a negative number
  - Benefits (receipts) = a positive number
- Because money is more valuable today than in the future, we need to describe cash receipts & disbursements at time they occur.

Example 3-1
Cash flows of 2 payment options

EXAMPLE 3-1

A machine will cost $30,000 to purchase. Annual operating and maintenance costs (O&M) will be $2000. The machine will save $10,000 per year in labor costs. The salvage value of the machine after 5 years will be $7000. Draw the cash flow diagram.



Example 3-1
Cash flows of 2 payment options

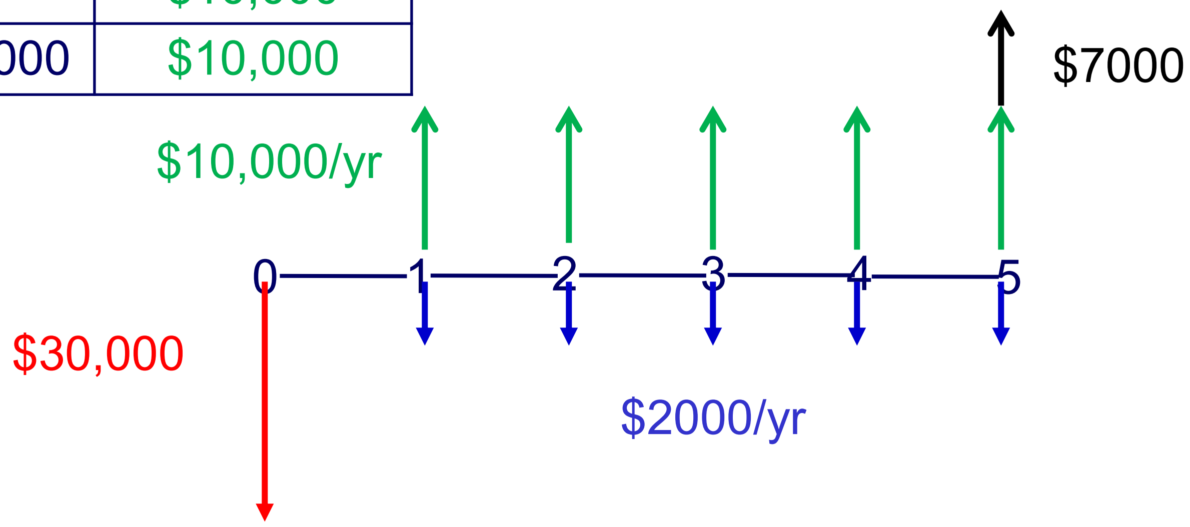
Purchase a new $30,000 machine,

- O&M costs = $2000/yr
- Savings = $10,000/yr
- Salvage value at Yr 5 = $7000

Draw the cash flow diagram

Example 3-1, Cash flows

End of Year	Costs & SV	Savings
0 (now)	−$30,000	$10,000
1	−2000	$10,000
2	−2000	$10,000
3	−2000	$10,000
4	−2000	$10,000
5	−2000+7000	$10,000

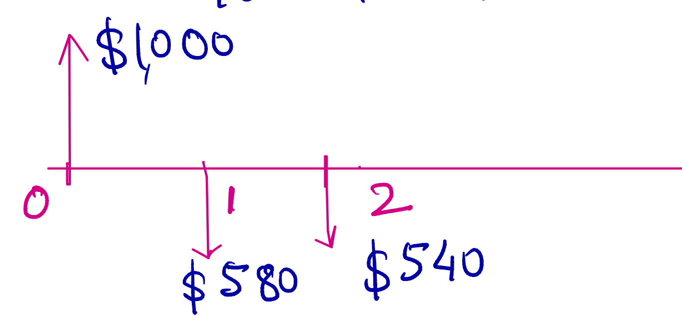


Example 3-2
Cash flow for repayment of a loan

EXAMPLE 3-2

A man borrowed $1000 from a bank at 8% interest. He agreed to repay the loan in two end-of-year payments. At the end of each year, he will repay half of the $1000 principal amount plus the interest that is due. Compute the borrower's cash flow.

Year	Principal	interest	Payment
0	$1000		
1	$1000	$\frac{8}{100} \times 1000 = 80$	$\frac{1000}{2} + 80 = 500 + 80 = \$580$
2	$500	$\frac{8}{100} \times 500 = 40$	$500 + 40 = \$540$

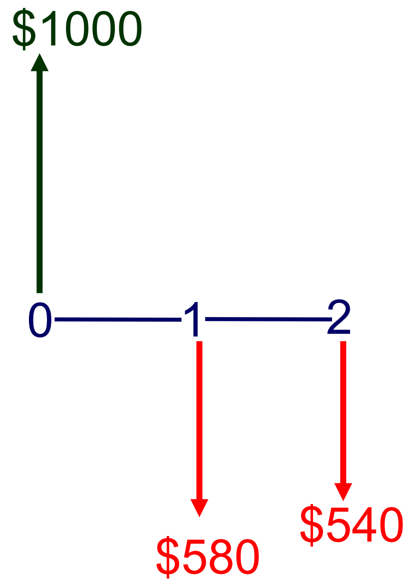


Example 3-2
Cash flow for repayment of a loan

To repay a loan of $1000 at 8% interest in 2 years

- Repay half of $1000 plus interest at the end of each year

Yr	Interest	Balance	Repayment	Cash Flow
0		1000		1000
1	80	500	500	−580
2	40	0	500	−540



Time Value of Money

Money has value

- Money can be leased or rented
- Payment is called interest
- If you put $1000 in a bank at 4% interest for one time period you will receive back your original $1000 plus $40

$1000 \times \frac{4}{100} = \$40$

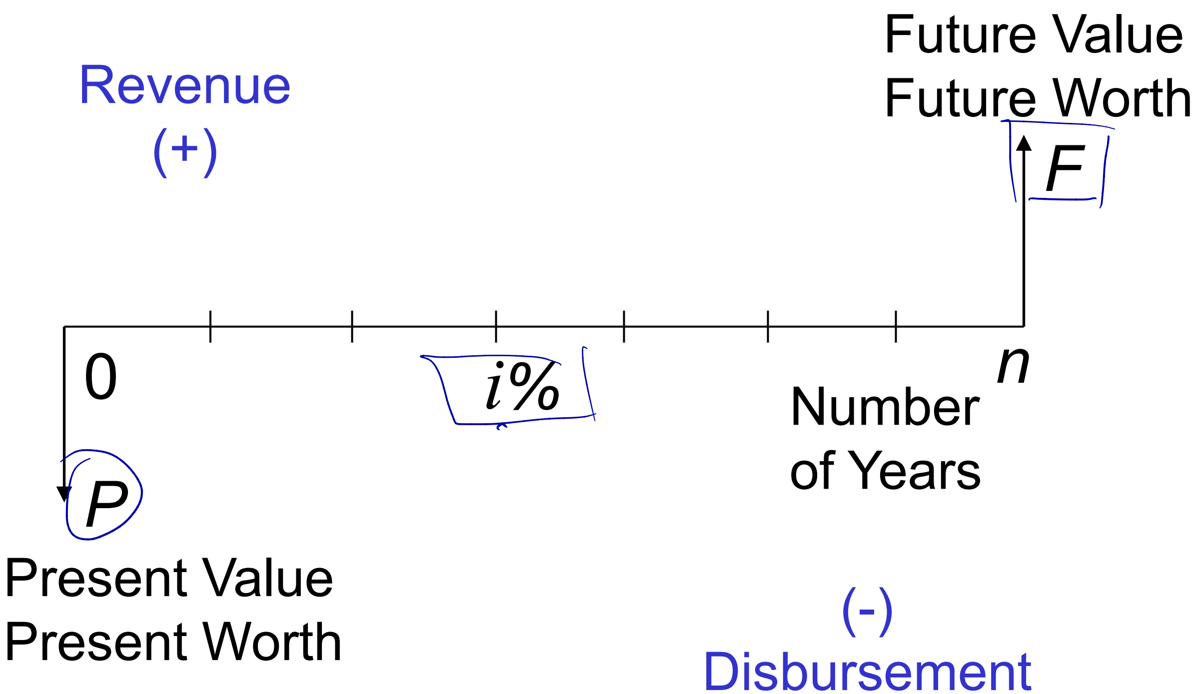
Original amount to be returned = $1000
Interest to be returned = $1000 x .04 = $40

Cash Flow Diagram

Invest P dollars at *i*% interest & receive F dollars after n years

$F = f(P, i\%, n)$

$P = f(F, i\%, n)$



Simple Interest on Loan

Is computed only on original sum—does not include interest earned or owed

P borrowed for *n* years

Total interest owed = $P \times i \times n$

- P = present sum of money
- i = interest rate
- n = number of periods (years)

Simple interest = \$1000 x .04/period x 2 periods = \$80

Example 3-3
Simple Interest Calculation

EXAMPLE 3-3

You have agreed to loan a friend $5000 for 5 years at a simple interest rate of 8% per year. How much interest will you receive from the loan? How much will your friend pay you at the end of 5 years?

P = $5,000
n = 5 years
i = 8% per year

Total interest = P x n x i
= 5000 x 5 x 8/100

Total interest = $2,000

Total money received after 5 years
= $5,000 + $2,000 = $7,000

Example 3-3
Simple Interest Calculation

Loan of $5000 for 5 yrs at simple interest rate of 8%

Total interest owed = $5000(8%)(5) = $2000

Amount due at end of loan = $5000 + 2000 = $7000

Compound Interest

\$100 at an interest rate of 10%

Year	Principal	Interest	Total
1	\$100	$10/100 \times 100 = 10$	$100 + 10 = \$110$
2	\$110	$\frac{10}{100} \times 110 = 11$	$100 + 11 = \$121$

interest using Simple interest

$$P \times n \times i = 100 \times 2 \times 10\% = 20$$

Total $= 100 + 20 = \$120$

P = Present Worth i% ⇒ interest rate
n ⇒ Total number of Period

Year	Principal	Interest	Total
1	P	$P \times i$	$P + Pi = P(1+i)$
2	$P(1+i)$	$P(1+i)i$	$P(1+i) + P(1+i)i$ $P(1+i)(1+i)$ $= P(1+i)^2$

3	$P(1+i)^2$	$P(1+i)^2 i$	$P(1+i)^2 + P(1+i)^2 i$ $P(1+i)^2(1+i)$ $\boxed{P(1+i)^3}$
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} i$	$P(1+i)^n$ ⇑ future Value

Compound Interest

- Interest computed on unpaid balance,
  - includes the principal
  - any unpaid interest from the preceding period

$$P = \frac{F}{(1+i)^n}$$

$$F = P(1+i)^n$$

Compound Interest on Loan

- Compound interest is computed on unpaid debt & unpaid interest
- Total interest earned = $P(1+i)^n - P$
  - Where
    - P = present sum of money
    - i = interest rate
    - n = number of periods (years)

Interest = $1000 × $(1+.04)^2$ − $1000 = $81.60

Compound Interest

For compound interest

$$F_1 = 5000(1 + 0.04)^1 = \$5200$$

$$F_2 = 5200(1 + 0.04)^1 = \$5408$$

$$F_3 = 5408(1 + 0.04)^1 = \$5624$$

Differences from simple interest magnify as # of periods & interest rates increase

Compound Interest

For compound interest

$$F_1 = P(1+i)$$

$$F_2 = F_1(1+i) = P(1+i)(1+i) = P(1+i)^2$$

$$F_3 = F_2(1+i) = P(1+i)^2(1+i) = P(1+i)^3$$

After n periods

$$F = P(1+i)^n$$

Which is true?

A. $P = F(1+i)^n$

B. $P = F(1+n)^i$

C. $P = F/(1+i)^n$

D. $P = F/(1+n)^i$

E. I don't know

Which is true?

A. $P = F(1+i)^n$

B. $P = F(1+n)^i$

C. $P = F/(1+i)^n$

D. $P = F/(1+n)^i$

E. I don't know

Example 3-4
Compound Interest Calculation

EXAMPLE 3-4

To highlight the difference between simple and compound interest, rework Example 3-3 using an interest rate of 8% per year compound interest. How will this change affect the amount that your friend pays you at the end of 5 years?

Original loan amount (original principal) = \$5000 = P

Loan term = 5 years = n

Interest rate charged = 8% per year compound interest = i

$$F = P(1+i)^n = 5000 \times (1+8\%)^5$$

$$F \simeq \$7,347$$

Example 3-4
Compound Interest Calculation

Loan of $5000 for 5 yrs at 8%

Year	Balance at the Beginning of the year	Interest	Balance at the end of the year
1	$5,000.00	$400.00	$5,400.00
2	$5,400.00	$432.00	$5,832.00
3	$5,832.00	$466.56	$6,298.56
4	$6,298.56	$503.88	$6,802.44
5	$6,802.44	$544.20	$7,346.64

Repaying a Debt
Plan #1: Constant Principal

Repay of a loan of $5000 in 5 yrs at interest rate of 8%

Plan #1: Constant principal payment plus interest due

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	$5,000.00	$400.00	$5,400.00	$400.00	$1,000.00	$1,400.00
2	$4,000.00	$320.00	$4,320.00	$320.00	$1,000.00	$1,320.00
3	$3,000.00	$240.00	$3,240.00	$240.00	$1,000.00	$1,240.00
4	$2,000.00	$160.00	$2,160.00	$160.00	$1,000.00	$1,160.00
5	$1,000.00	$80.00	$1,080.00	$80.00	$1,000.00	$1,080.00
	Subtotal			$1,200.00	$5,000.00	$6,200.00

Repaying a Debt
Plan #2: Interest Only

Repay of a loan of $5000 in 5 yrs at interest rate of 8%

Plan #2: Annual interest payment & principal payment at end of 5 yrs

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	$5,000.00	$400.00	$5,400.00	$400.00	$0.00	$400.00
2	$5,000.00	$400.00	$5,400.00	$400.00	$0.00	$400.00
3	$5,000.00	$400.00	$5,400.00	$400.00	$0.00	$400.00
4	$5,000.00	$400.00	$5,400.00	$400.00	$0.00	$400.00
5	$5,000.00	$400.00	$5,400.00	$400.00	$5,000.00	$5,400.00
	Subtotal			$2,000.00	$5,000.00	$7,000.00

Repaying a Debt
Plan #3: Constant Payment

Repay of a loan of $5000 in 5 yrs at interest rate of 8%

Plan #3: Constant annual payments

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	$5,000.00	$400.00	$5,400.00	$400.00	$852.28	$1,252.28
2	$4,147.72	$331.82	$4,479.54	$331.82	$920.46	$1,252.28
3	$3,227.25	$258.18	$3,485.43	$258.18	$994.10	$1,252.28
4	$2,233.15	$178.65	$2,411.80	$178.65	$1,073.63	$1,252.28
5	$1,159.52	$92.76	$1,252.28	$92.76	$1,159.52	$1,252.28
	Subtotal			$1,261.41	$5,000.00	$6,261.41

Repaying a Debt
Plan #4: All at Maturity

Repay of a loan of $5000 in 5 yrs at interest rate of 8%
Plan #4: All payment at end of 5 years

Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	$5,000.00	$400.00	$5,400.00	$0.00	$0.00	$0.00
2	$5,400.00	$432.00	$5,832.00	$0.00	$0.00	$0.00
3	$5,832.00	$466.56	$6,298.56	$0.00	$0.00	$0.00
4	$6,298.56	$503.88	$6,802.44	$0.00	$0.00	$0.00
5	$6,802.44	$544.20	$7,346.64	$2,346.64	$5,000.00	$7,346.64
	Subtotal			$2,346.64	$5,000.00	$7,346.64

4 Repayment Plans

- Differences:
  - Repayment structure (repayment amounts at different times)
  - Total payment amount
- Similarities:
  - All interest charges were calculated at 8%
  - All repaid a $5000 loan in 5 years

Equivalence

- If a firm believes 8% was reasonable, it would have no preference about whether it received $5000 now or was paid by any of the 4 repayment plans.
- The 4 repayment plans are <u>equivalent</u> to one another & to $5000 now at 8% interest

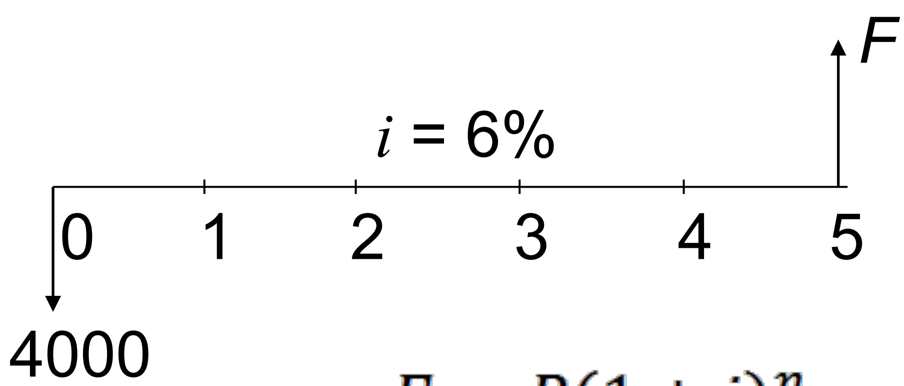
Use of Equivalence in Engineering Economic Studies

- Using equivalence, one can convert different types of cash flows at different points of time to an equivalent value at a common reference point
- Equivalence depends on interest rate

Example

If you were to receive \$4000 today to invest at 6% interest, what would this be equivalent to in 5 years?

Given: $P = 4000$, $i = 6\%$, $n = 5$



$$F = P(1+i)^n$$
$$F = 4000(1+0.06)^5 = \$5352.90$$

You deposit $100 in account earning 5%

After 4 years the value in account is

A. –$121.55
B. $121.55
C. $121.67
D. $431.01
E. None of the above

$$F = P(1 + i)^n$$

You deposit $100 in account earning 5%.

After 4 years the value in account is

A. –$121.55
B. $121.55 $F = 100(1.05)^4$
C. $121.67
D. $431.01
E. None of the above

Interest Formulas

Notation:

i = Interest rate per interest period

n = Number of interest periods

P = Present sum of money (Present worth)

F = Future sum of money (Future worth)

Basic factors

Equation: $P = F/(1+i)^n$

Factor: $P = F(P/F, i, n)$

Function: =PV(rate, nper, pmt, [FV], [type])

Equation: $F = P(1+i)^n$

Factor: $F = P(F/P, i, n)$

Function: =FV(rate, nper, pmt, [PV], [type])

Factors & Functions

Variable	Engineering Economy	Spreadsheets
Present value	*P*	PV
Future value	*F*	FV
Uniform series	*A*	PMT
Interest rate	*i*	RATE
Number of periods	*n*	NPER

Notation for Calculating a Future Value

Formula:

$F = P(1+i)^n$ is the

<u>*single payment compound amount factor*</u>

Functional notation:

$F = P(F/P, i, n)$ $F = 5000(F/P, 6\%, 10)$

$F = \not{P}(F/\not{P})$ is dimensionally correct

In Excel,

=FV(rate,nper,pmt,[pv],[type])

Notation for Calculating a Present Value

$$P = F\left(\frac{1}{1+i}\right)^n = \frac{F}{(1+i)^n}$$ is the

Single payment present worth factor

Functional notation:

$$P = F(P/F, i, n) \qquad P = 5000(P/F, 6\%, 10)$$

In Excel,

=PV(rate,nper,pmt,[fv],[type])

Excel financial functions

=PV(rate, nper, pmt, [fv], [type])

=FV(rate, nper, pmt, [pv],[type])

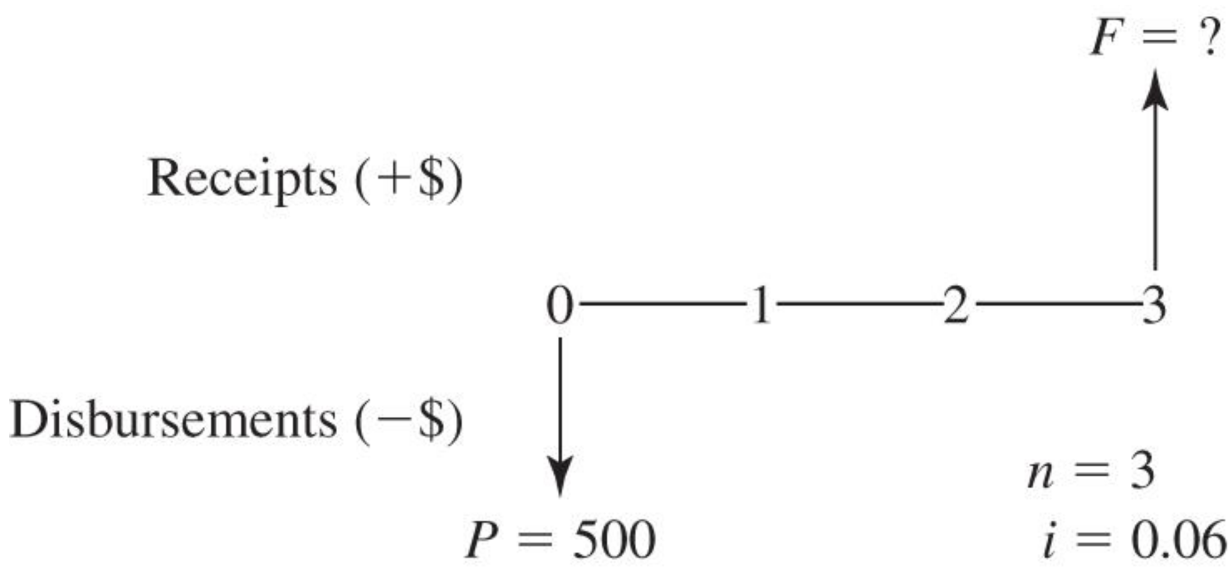
=PMT(rate, nper, pv,[fv],[type])

=NPER(rate, pmt, pv, [fv], [type])

=RATE(nper, pmt, pv, [fv], [type],[guess])

Example 3-5

$500 is deposited today. What is it worth in 3 years at 6% interest?



Example 3-5

- $F = P(1+i)^n$

  $= 500(1+.06)^3 = 500(1.191) = \$595.51$

- $F = P(F/P)$

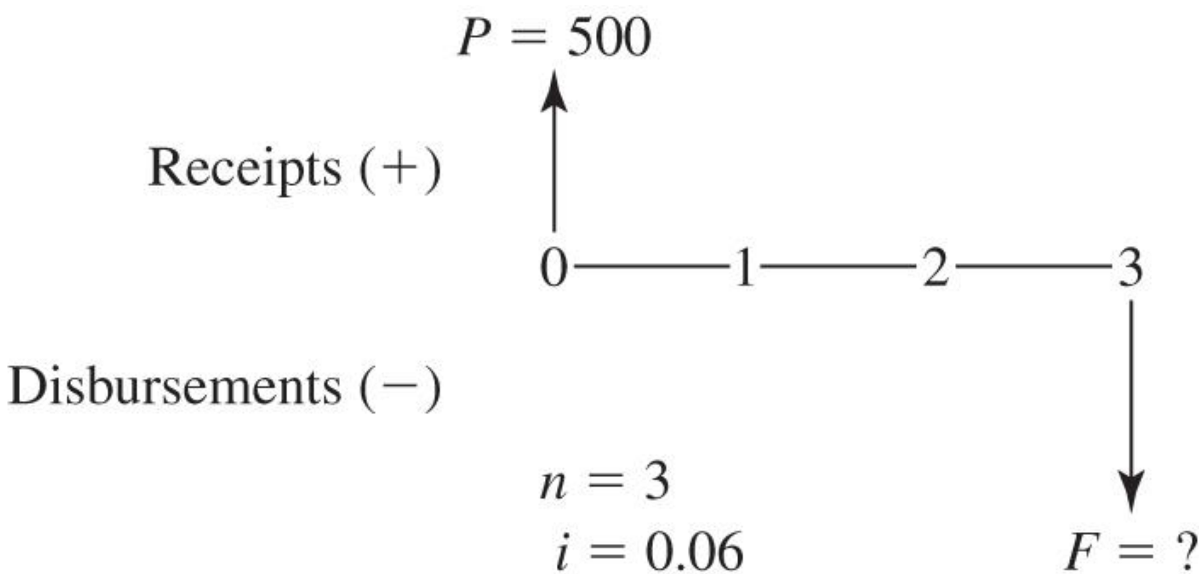
  $= 500(F/P,6\%,3) = 500(1.191) = \$595.50$

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	3-5	6%	3	0	-500		$595.51	=FV(B2,C2,D2,E2)

=FV(rate, nper, pmt, [pv], [type])

Example 3-5

From the bank's point of view, are the numbers different? No—only the sign changes.



How Excel computes this

=FV(rate,nper,pmt,pv)

i = 6% nper = 3   pmt = 0   pv = 500

FV = −595.508

Excel uses the following equation:

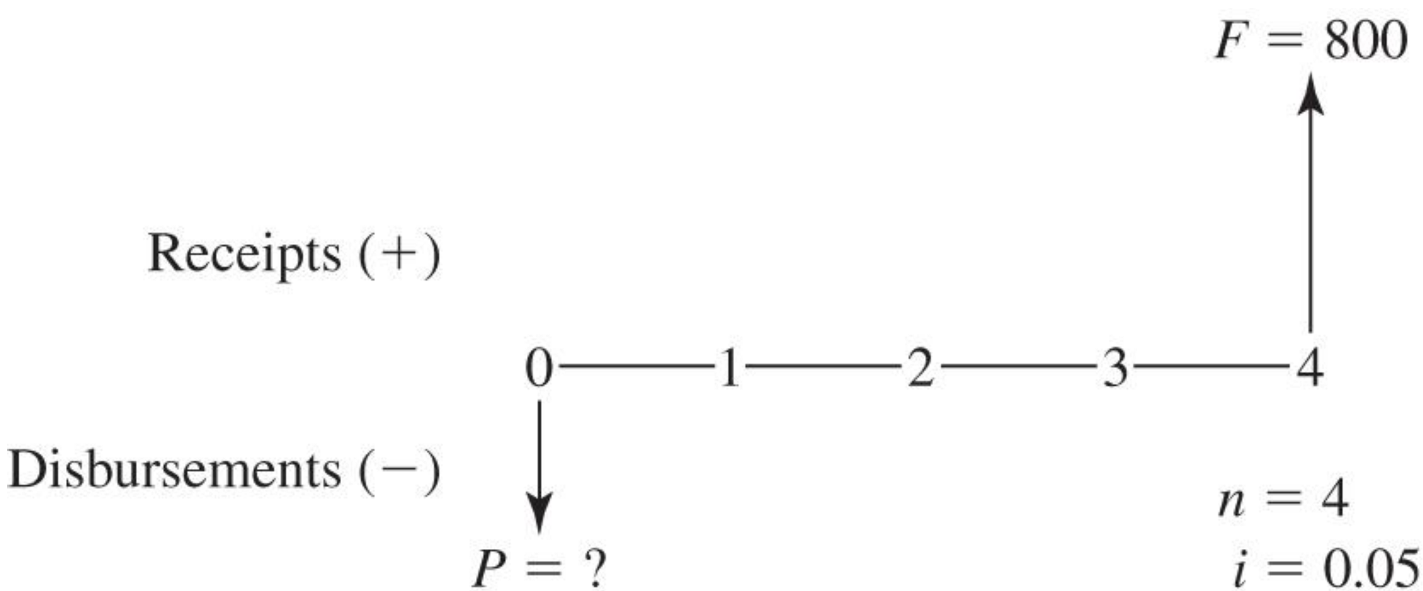
$$PMT\left[\frac{1-(1+i)^{-n}}{i}\right]+FV(1+i)^{-n}+PV=0$$

So PMT, FV, & PV cannot be the same sign

Example 3-6

EXAMPLE 3-6

If you wish to have $800 in a savings account at the end of 4 years, and 5% interest will be paid annually, how much should you put into the savings account now?



Example 3-6

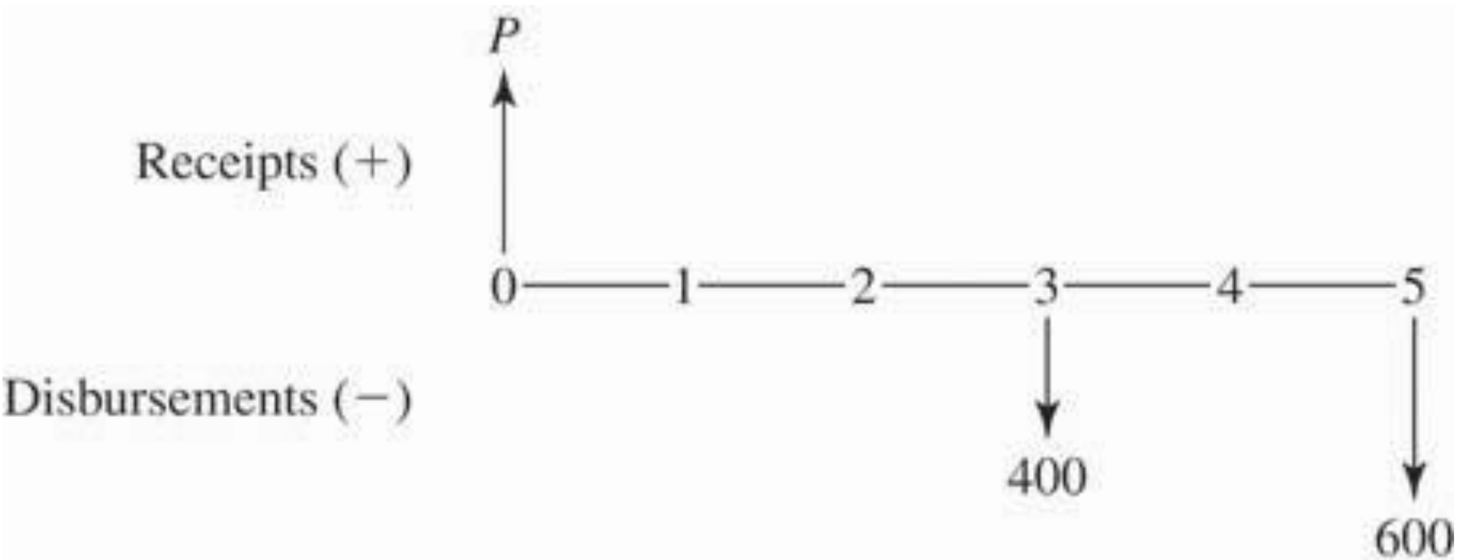
$$P = F / (1+i)^n = 800/(1+0.05)^{-4} = \$658.16$$

$$P = F(P/F, i, n) = 800(P/F, 5\%, 4) = 800(0.8227) = 658.16$$

or

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	3-6	5%	4	0		800	-$658.16	=PV(B2,C2,D2,F2)
3								=PV(rate,nper,pmt,[fv],[type])

2 Cash Outflows



i = 12%

2 Cash Outflows

$$P = 400(P/F, 12\%, 3) + 600(P/F, 12\%, 5)$$

$$= 400(0.7118) + 600(0.5674) = \$625.16$$

or

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	2 CF	12%	3	0		-400	$284.71	=PV(B2,C2,D2,F2)
3		12%	5	0		-600	$340.46	=PV(B3,C3,D3,F3)
4							$625.17	=G2+G3

You deposit $100 in an account earning 5%

After 4 years the value in the account is:

A. −121.55
B. 121.55
C. 121.66
D. −121.66

A. Something else or I don't know

You deposit $100 in an account earning 5%

After 4 years the value in the account is:

A. −121.55

B. 121.55

$$= 100(F/P, 5\%, 4) = 100(1.216)$$

$$= FV(5\%, 4, 0, -100)$$

C. 121.66

D. −121.66

E. Something else or I don’t know

You need $6000 in 3 years as a down payment on a car.

If your savings earn 0.25% interest per month, how much do you need to deposit today to have $6000 in 3 years?

A. 5955.22
B. 5490.85
C. 5484.20
D. 2070.19
E. I don’t know

You need $6000 in 3 years as a down payment on a car.

If your savings earn 0.25% interest per month, how much do you need to deposit today to have $6000 in 3 years?

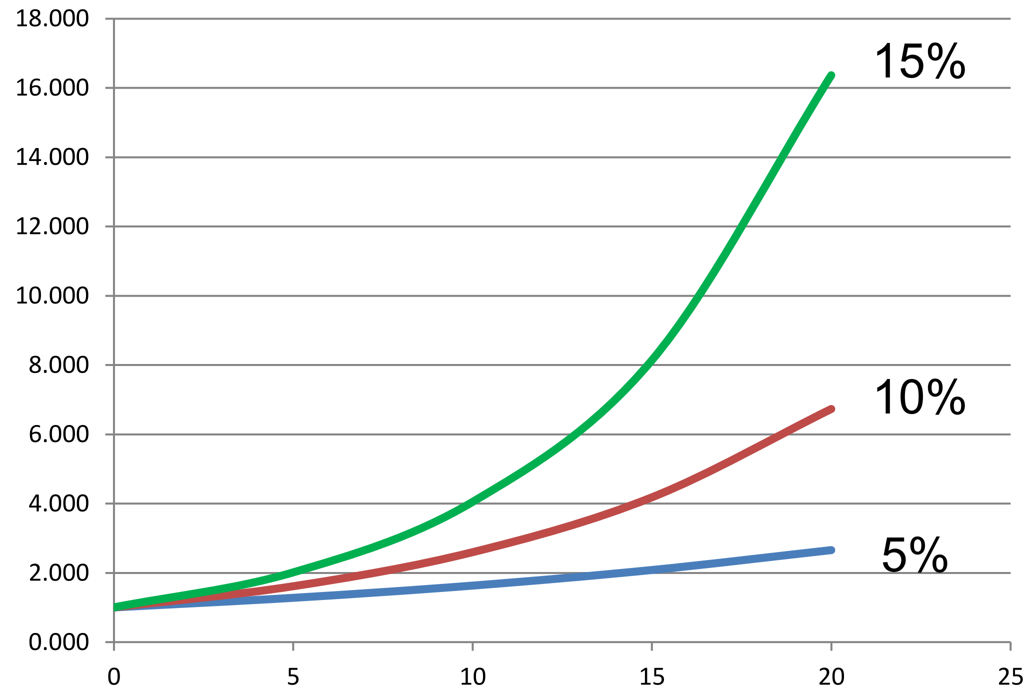
A. 5955.22
B. 5490.85
C. 5484.20
D. 2070.19
E. I don’t know

$$= 6000(P/F, 0.25\%, 36) = 6000(0.9140)$$
$$= PV(0.25\%, 36, 0, -6000)$$

Example 3-7 Single Payment Compound Interest Formulas

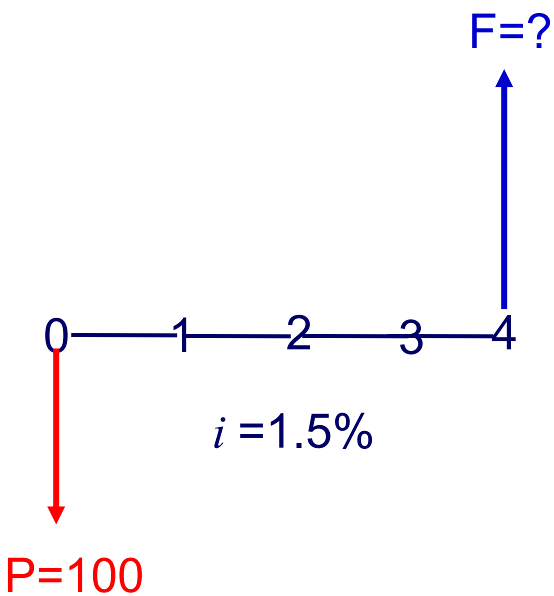
Tabulate the future value factor for interest rates of 5%, 10%, & 15% for n's from 0 to 20 (in 5's).

n	5%	10%	15%
0	1.000	1.000	1.000
5	1.276	1.611	2.011
10	1.629	2.594	4.046
15	2.079	4.177	8.137
20	2.653	6.727	16.367



Example 3-8 Single Payment Compound Interest Formulas

$100 were deposited in a saving account (pays 6% compounded quarterly) for 1 year



i_{qtr} =1.5%, n = 4 quarters

$$F = P(1+i)^n = 100(1+0.015)^4$$
$$= \$106.14$$

$$F = P(F/P, i, n) = 100(F/P, 1.5\%, 4)$$
$$= 100(1.061) = \$106.10$$

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	3-8	1.5%	4	0	-100		$106.14	=FV(B2,C2,D2,E2)

Nominal & Effective Interest

- Nominal interest rate/year: the annual interest rate w/o considering the effect of any compounding.
  - 12%/year

- Interest rate/period: the nominal interest rate/year divided by the number of interest compounding periods.
  - (12%/year)/(12 months/year) = 1%/month

Effective Interest Rate

The *effective interest rate* is given by the formula:

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

where *r* = nominal annual interest rate

m = number of compounding periods per year

Example 3-9 Nominal & Effective Interest Rates

If a credit card charges 1.5% interest every month, what are the nominal & effective interest rates per year?

$$r = 12 \times 1.5\% = 18\%$$

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

$$i_a = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 0.1956 = 19.56\%$$

	A	B	C	D	E
1	Nominal annual rate, r	Periods per year, m	Effective rate, i_a	Answer	Spreadsheet Function
2	18.0%	12		19.56%	=EFFECT(A2,B2)

Example 3-10 Application of Nominal & Effective Interest Rates

"If I give you $100 today, you will write me a check for $120, which you will redeem or I will cash on your next payday in 2 weeks."

$$\text{Bi-weekly interest rate} = (\$120 - 100)/100 = 20\%$$

$$\text{Nominal annual rate} = 20\% * 26 = 520\%$$

$$i_a = \left(1 + \frac{5.20}{26}\right)^{26} - 1 = 113.48 = 11{,}348\%$$

$$\text{End of year balance owed} = \$100 \text{ principal} + \$11{,}348 \text{ interest}$$

$$F = P(1+i)^n = 100(1+0.20)^{26} = \$11{,}448$$

A credit card's APR is 12% with monthly compounding

What is the effective interest rate?

A. 12.00%
B. 14.4%
C. 4.095%
D. 12.68%
E. None of the above

A credit card's APR is 12% with monthly compounding

What is the effective interest rate?

A. 12.00%
B. 14.4%
C. 4.095%
D. 12.68%
E. None of the above

$$= \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

Example 3-11 Application of Continuous Compounding

6% interest compounded continuously.

$$\text{Effective interest rate} = e^{r} - 1$$
$$= e^{0.06} - 1 = 0.0618$$
$$= 6.18\%$$