Engineering Economic Analysis

FOURTEENTH EDITION

Chapter 4

Equivalence for Repeated Cash Flows

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Chapter Outline

- Uniform Series Compound Interest Formulas
- Cash Flows That Do Not Match Basic Patterns
- Economic Equivalence Viewed as a Moment Diagram
- Relationships Between Compound Interest Factors
- Arithmetic Gradient
- Geometric Gradient
- Spreadsheets for Economic Analysis
- Compounding Period & Payment Period Differ

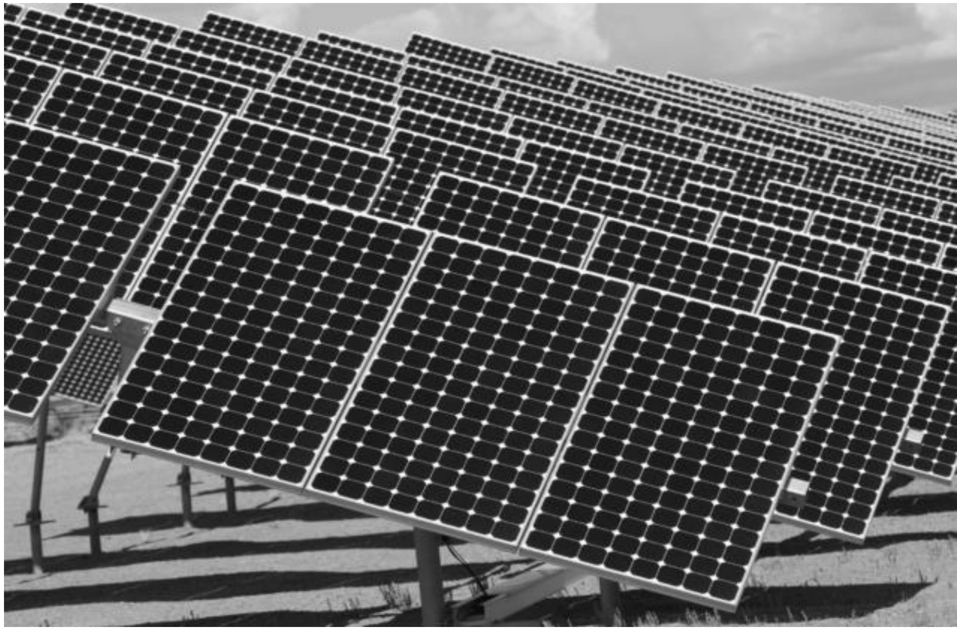
Learning Objectives

- Solve problems using uniform series compound interest formulas
- Use arithmetic & geometric gradients in modeling economic analysis
- Understand why cash flows assume uniformity
- Use spreadsheet to model & solve economic analysis problems

Vignette: Student Solar Power

Indiana State University (ISU) mechanical & manufacturing engineering technology students designed a photovoltaic system to make use of solar energy in 2008.

- 2-axis tracking system
- 4 PV panels of 123 watts each, life of 25 yrs..
- Most electrical parts provided free by the college CIM Lab.

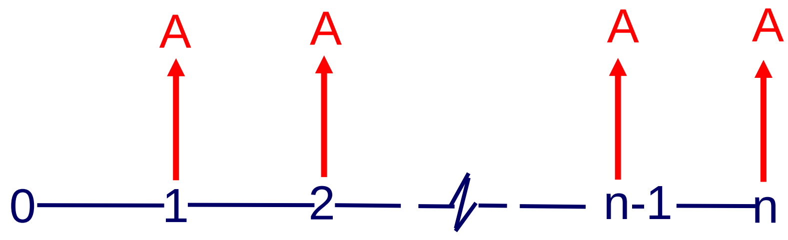


Vignette: Student Solar Power

1. Panels were purchased by ISU 5 years ago. Is the purchase cost a sunk cost?
2. How much difference due to a city's longitude if same panel installed there?
3. How important are latitude & yearly days of sunshine in system economics?
4. What costs must be considered & how can they be estimated over time?
5. How to compute the annual savings? Do panels decline in efficiency each year?

Uniform Series Compound Interest Formulas

A = end of period cash flow in a uniform series



Examples:

- Automobile loans, mortgage payments, insurance premium, rents, & other periodic payments
- Estimated future costs & benefits

Uniform Series Compound Interest Formulas

Uniform Series Compound Amount Factor

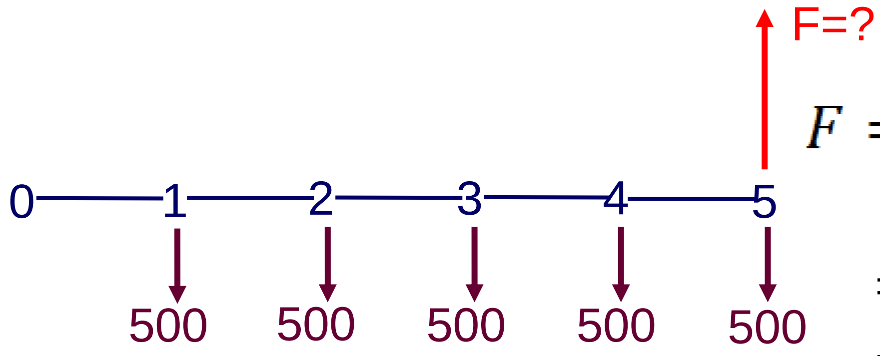
$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i, n) \qquad (4\text{-}4)$$

Uniform Series Sinking Fund Factor

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = F(A/F, i, n) \qquad (4\text{-}5)$$

Example 4-1 Uniform Series Compound Interest Formulas

$500 deposited in a credit union (pays 5% compounded annually) at the end of each year for 5 years, how much do you have after the 5th deposit?



$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i, n)$$

$$= 500(F/A, 5\%, 5) = 500(5.526)$$

$$= \$2763$$

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	4-1	5%	5	-500	0		$2,762.82	=FV(B2,C2,D2,E2)

Example 4-2 Uniform Series; Multiple Cash Flows

Initial deposit = \$685; \$375 deposited monthly. Interest rate = 6%, monthly compounding. How much is saved after 48 months?

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	4-2	0.5%	48	-375	-685		\$21,156.97	=FV(B2,C2,D2,E2)

$$F = 375(F/A, 0.5\%, 48) + 685(F/P, 0.5\%, 48) = \$21{,}156.7$$

You deposit $200 now in account earning 3%.

After 5 years the value in account is

A. $206.00
B. $231.85
C. $218.00
D. –$231.85
E. None of the above

You deposit $200 in account earning 3%.

After 5 years the value in account is

A. $206.00
B. $231.85
C. $218.00
D. –$231.85
E. None of the above

$$= 200(F/P, 3\%, 5) = 200(1.159)$$
$$= FV(3\%, 5, 0, -200)$$

You deposit $200 at end of each year in account earning 6%.

After 5 years the value in account is

A. −$267.65
B. $1060
C. $1127.42
D. $1360.38
E. None of the above

You deposit \$200 at end of each year in account earning 6%.

After 5 years the value in account is

A. −\$267.65
B. \$1060
C. \$1127.42
D. \$1360.38
E. None of the above

$= 200(F/A, 6\%, 5) = 200(5.637)$

$= FV(6\%, 5, -200)$

Example 4-3

How much must Jim deposit at the end of each month to get \$1000 at year end? Bank pays 6% interest compounded monthly.

$$i_{mo} = {}^{6\%}/_{12} = 0.5\%$$

$$A = F(A/F, i, n) = 1000(A/F, 0.5\%, 12)$$
$$= 1000(0.0811) = \$81.10$$

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	4-3	0.5%	12		0	1000	-\$81.07	=PMT(B2,C2,E2,F2)

Uniform Series Compound Interest Formulas

Uniform Series Capital Recovery Factor

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P(A/P, i, n) \qquad (4\text{-}6)$$

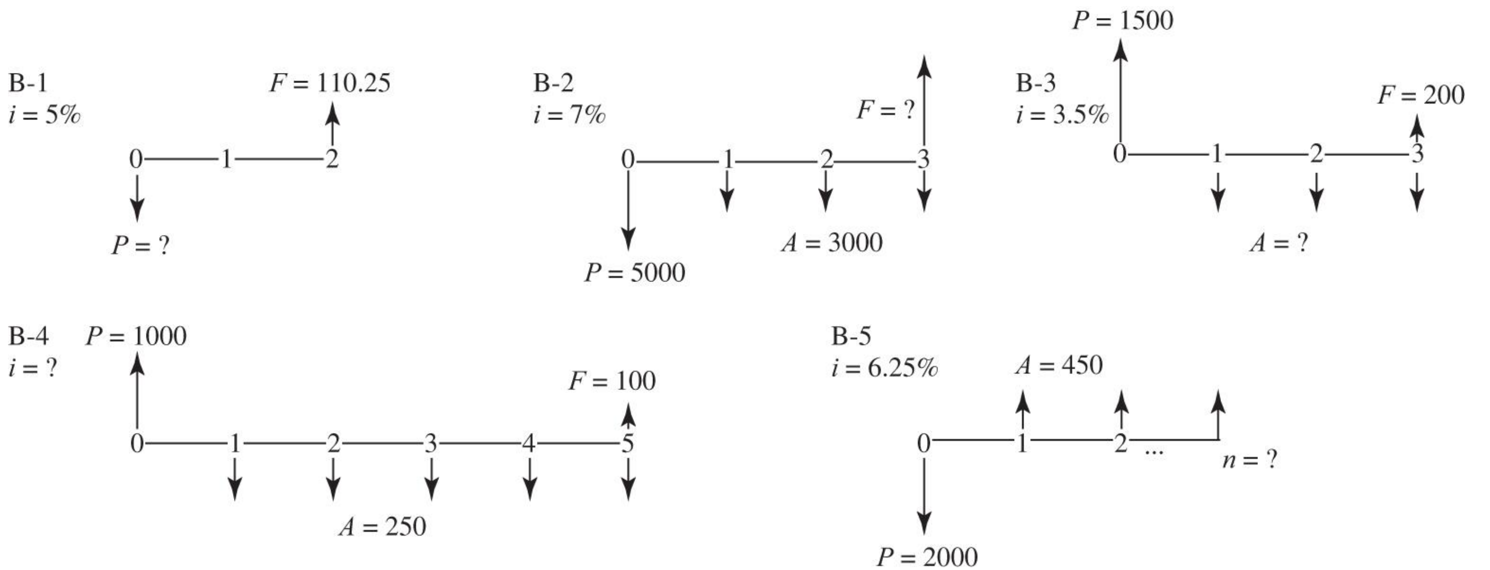
Uniform Series Present Worth Factor

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A, i, n) \qquad (4\text{-}7)$$

The Annuity Functions

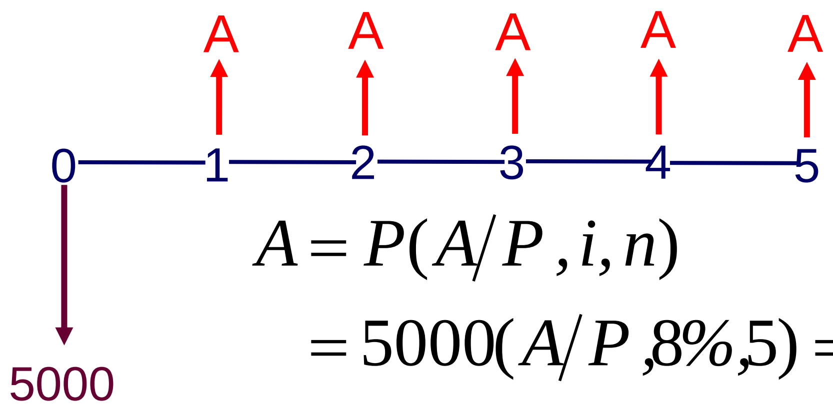
See Appendix B

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	B-1	5.0%	2	0		110.25	PV	-$100.00	=PV(B2,C2,D2,F2)
3	B-2	7.0%	3	-3000	-5000		FV	$15,770	=FV(B3,C3,D3,E3)
4	B-3	3.5%	3		1500	200	PMT	-$599.79	=PMT(B4,C4,E4,F4)
5	B-4		5	-250	1000	100	RATE	5.15%	=RATE(C5,D5,E5,F5)
6	B-5	6.25%		450	-2000	0	N	5.37	=NPER(B6,D6,E6,F6)



Example 4-4

A machine costs $5000 & lasts 5 years. If interest is 8%, how much must be saved annually to recover the investment?



$$A = P(A/P, i, n)$$

$$= 5000(A/P, 8\%, 5) = 5000(0.2505) = \$1252.50$$

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	4-4	8.0%	5		-5000	0	$1,252.28	=PMT(B2,C2,E2,F2)

To have $1M after 40 years in account earning 6%

Your annual deposit must be

A. $6096
B. $25,000
C. $12,649
D. $6462
E. None of the above

To have $1M after 40 years in account earning 6%

Your annual deposit must be

A. $6096

B. $25,000

C. $12,649

D. $6462

E. None of the above

$$= 1M(A/F, 6\%, 40) = 1M(0.00646)$$

$$= PMT(6\%, 40, 0, -1000000)$$

Example 4-5

A machine costs $30,000, O&M = $2000/yr. Savings = $10,000/yr. Salvage @ 5 yrs = $7000. *i* = 10%. PW = ??

$$P = -30{,}000 + (10{,}000 - 2000)(P/A, 10\%, 5) + 7000(P/F, 10\%, 5)$$
$$= -30{,}000 + (8{,}000)(3.791) + 7000(0.6209) = \$4672$$

	A	B	C	D	E	F	G	H
1	ID	*i*	*n*	*PMT*	*PV*	*FV*	Answer	Formula
2	4-5	10%	5	8000		7000	-$34,672.74	=PV(B2,C2,D2,F2)
3							$34,672.74	-G2
4					-30,000		4,672.74	=+G3+E4

Example 4-6, Find Rate of Return

A machine costs $30,000, O&M = $2000/yr. Savings = $10,000/yr. Salvage @ 5 yrs = $7000. ROR= ??

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-6		5	8000	-30000	7000	RATE	15.38%	=RATE(C2,D2,E2,F2)

$$0 = -30{,}000 + (10{,}000 - 2000)(P/A, i, 5) + 7000(P/F, i, 5)$$

To solve with tabulated factors assume the interest rate & see if the PW = 0

Example 4-6, Find Rate of Return

Try 15%

$$P_{15} = -30{,}000 + (10{,}000 - 2000)(P/A, 15\%, 5) + 7000(P/F, 15\%, 5)$$

$$P_{15} = -30{,}000 + (8000)(3.352) + 7000(.4972) = \$296.4$$

Try 18%

$$P_{18} = -30{,}000 + (10{,}000 - 2000)(P/A, 18\%, 5) + 7000(P/F, 18\%, 5)$$

$$P_{15} = -30{,}000 + (8000)(3.127) + 7000(.4371) = -\$1924.3$$

By interpolation, i = 15.4%

Interpolation

	Interest Rate		Present Worth
b, a	15%	d, c	\$296.40
	i		0
	18%		−\$1924.30

(a = 15% to i; b = 15% to 18%; c = \$296.40 to 0; d = \$296.40 to −\$1924.30)

Using ratios, $\dfrac{a}{b} = \dfrac{c}{d}$

$$\frac{i - 15\%}{18\% - 15\%} = \frac{0 - 296.40}{-1924.30 - 296.40}$$

$$\frac{i - 15\%}{3\%} = \frac{-296.4}{-2220.7} = \frac{296.4}{2220.7}$$

$$i = 15\% + \frac{3\% \times 296.4}{2220.7} = 15.4\%$$

Find the value of x using interpolation

Interest rate	**Value**
2%	10.950
3%	x
4%	12.006

A. 11.5
B. 11.464
C. 11.478
D. I don't know

Find the value of x using interpolation

Interest rate	Value
2%	10.950
3%	x
4%	12.006

A. 11.5
B. 11.464
C. 11.478
D. I don't know

$$\frac{3\% - 2\%}{4\% - 2\%} = \frac{x - 10.950}{12.006 - 10.950}$$

The firm invests $75,000 to save $9000/year in energy costs for 15 yrs

What is the project's rate of return?

A. 8.44%
B. 0.08%
C. 8%
D. 9.36%
E. I don't know

The firm invests $75,000 to save $9000/year in energy costs for 15 yrs

What is the project's rate of return?

A. 8.44% $= RATE(15{,}9000,-75000)$
B. 0.08%
C. 8%
D. 9.36%
E. I don't know

Example 4-7, Effective Rates

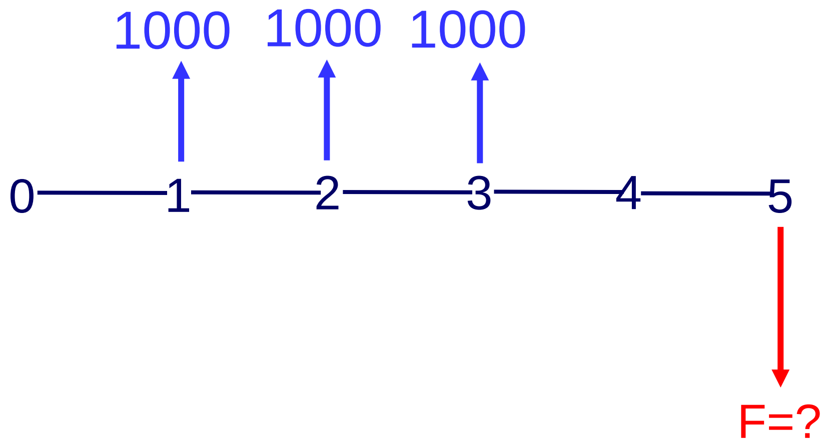
New car costs $15,732; 48 monthly payments of $398. What is monthly interest rate? Effective annual rate?

	A	B	C	D	E	F	G	H	I
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer	Formula
2	Exp. 4-7		48	-398	15,732	0	*i*	0.822%	=RATE(C2,D2,E2,F2)
3		monthly						annual	
4	Effective	0.822%	12	0	-1			1.1032	=FV(B4,C4,D4,E4)
5	or						i_a	10.33%	=FV-1
6	Nominal	0.822%	12				r	9.86%	=B6*C6
7	Effective	9.86%	12				i_a	10.32%	=EFFECT(B7,C7)

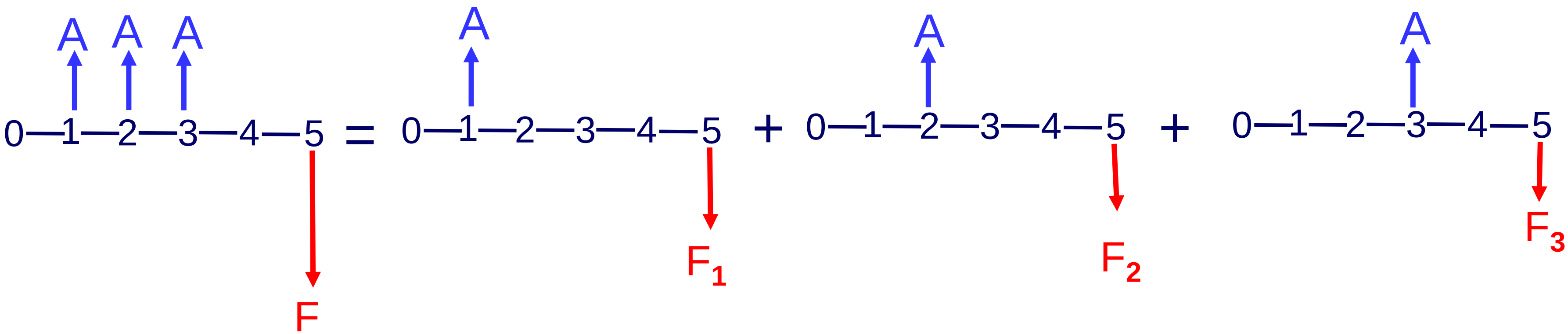
Example 4-8

A student is borrowing $1000/yr for 3 years. The loan will be repaid 2 years later at 15% interest rate. Find *F*.

Year	Cash Flow
1	+1000
2	+1000
3	+1000
4	0
5	-F



Example 4-8, Solution #1

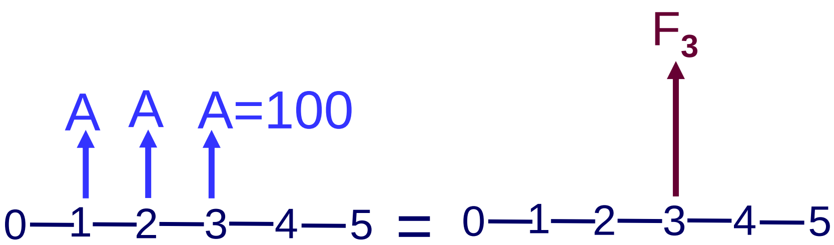


$$F = F_1 + F_2 + F_3$$

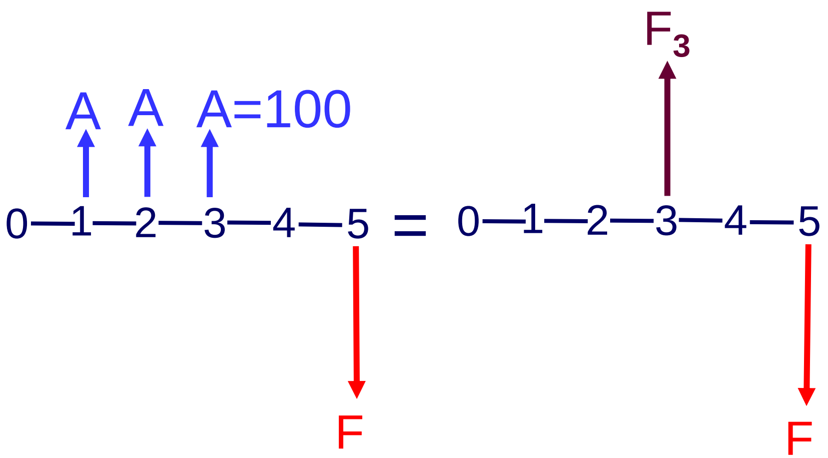
$$= 1000(F/P, 15\%, 4) + 1000(F/P, 15\%, 3) + 1000(F/P, 15\%, 2)$$

$$= 1000(1.749) + 1000(1.521) + 1000(1.322) = \$4592$$

Example 4-8, Solution #2



$$F_3 = 1000(F/A, 15\%, 3)$$
$$= 1000(3.472) = \$3472$$



$$F = F_3(F/P, 15\%, 2)$$
$$= 3472(1.322) = \$4590$$

or

$$F = 1000(F/A, 15\%, 3)(F/P, 15\%, 2)$$
$$= 1000(3.472)(1.322) = \$4590$$

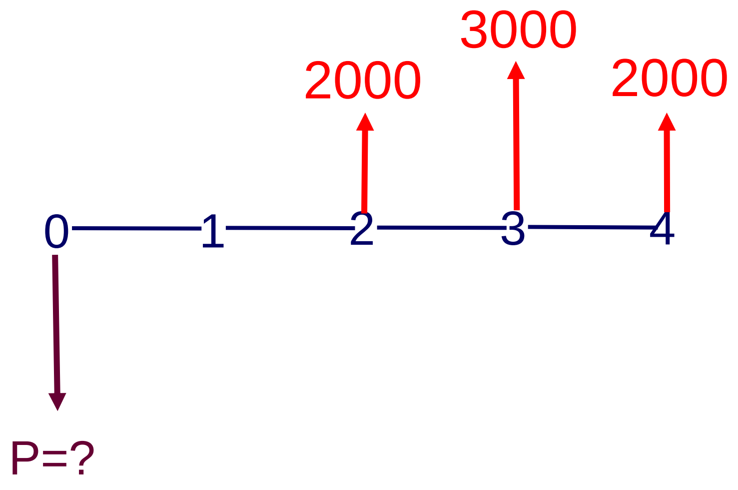
Example 4-8, Solution #3

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 4-8	15%	3	1000	0		F_3	-$3,472.50
3		15%	2	0	-3472.50		F	$4,592.38
4					=-H2			

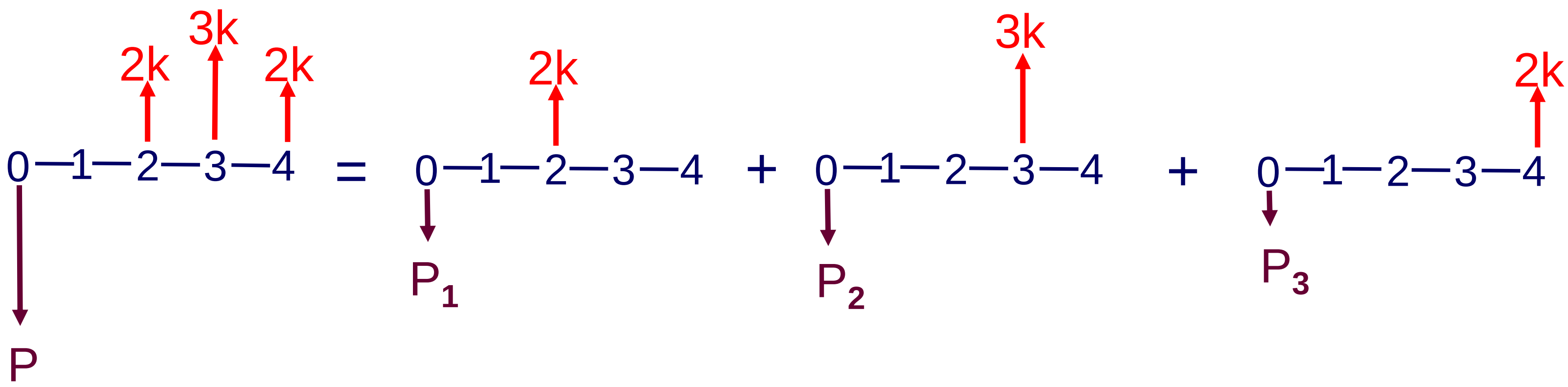
Example 4-9

What must be deposited in a saving account paying 15% interest, to support 3 later withdrawals?

Year	Cash Flow
0	-P
1	0
2	+2000
3	+3000
4	+2000

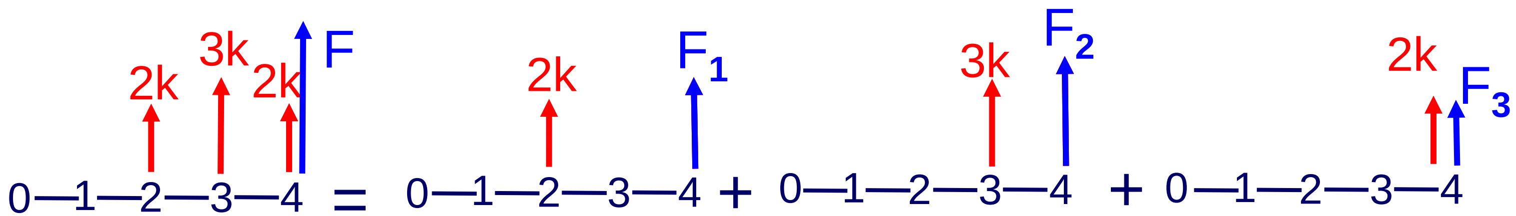


Example 4-9, Solution #1

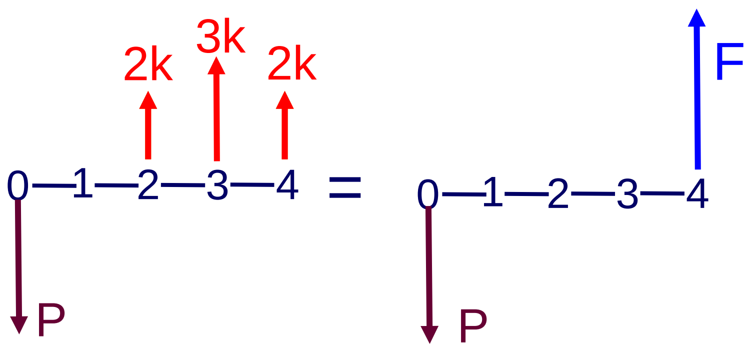


$$P = P_1 + P_2 + P_3$$
$$= 2000(P/F,15\%,2) + 3000(P/F,15\%,3) + 2000(P/F,15\%,4)$$
$$= 2000(0.7561) + 3000(0.6575) + 2000(0.5718) = \$4628$$

Example 4-9, Solution #2

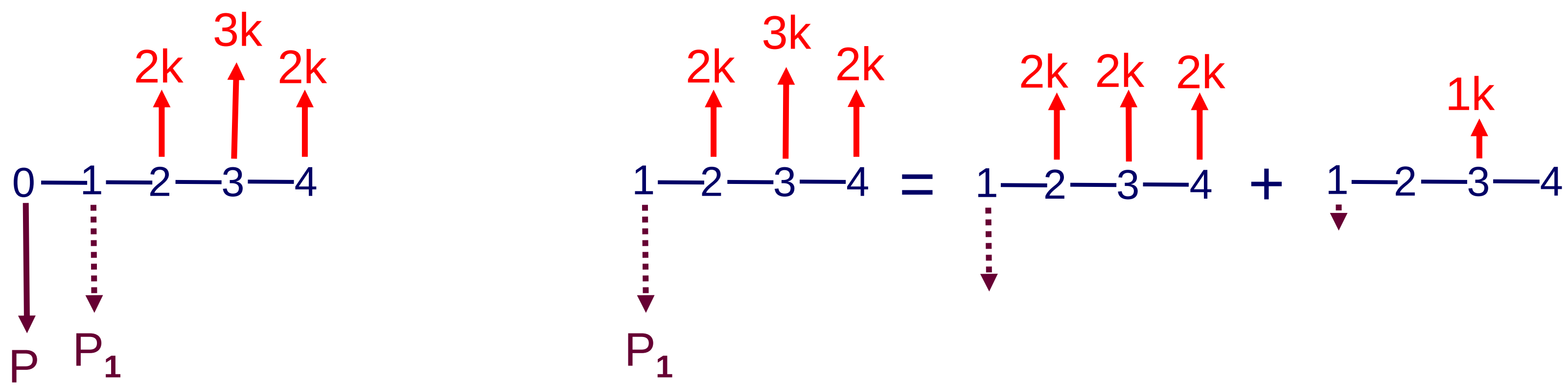


$$F = F_1 + F_2 + F_3$$
$$= 2000(F/P,15\%,2) + 3000(F/P,15\%,1) + 2000$$
$$= 2000(1.322) + 3000(1.150) + 2000 = \$8094$$



$$P = F(P/F,15\%,4)$$
$$= 8094(0.5718) = \$4628$$

Example 4-9, Solution #3



$$P = P_1\,(P/F,15\%,1)$$
$$= [2000(P/A,15\%,3) + 1000(P/F,15\%,2)](P/F,15\%,1)$$
$$= [2000(2.283) + 1000(0.7561)](0.8696) = \$4628$$

Example 4-9, Solution #4

	A	B	C	D	E	F	G	H
1	Problem	*i*	*n*	*PMT*	*PV*	*FV*	Solve for	Answer
2	Exp. 4-9	15%	2	0		2000	P_1	-$1,512.29
3		15%	3	0		3,000	P_2	-$1,972.55
4		15%	4	0		2,000	P_3	-$1,143.51
5							P	-$4,628.34

Relationships Between Compound Interest Factors

Single Payment

$$(F/P,i,n) = \frac{1}{(P/F,i,n)} \quad (4\text{-}8)$$

Uniform Series

$$(A/P,i,n) = \frac{1}{(P/A,i,n)} \quad (4\text{-}9)$$

$$(F/A,i,n) = \frac{1}{(A/F,i,n)} \quad (4\text{-}10)$$

Relationships Between Compound Interest Factors

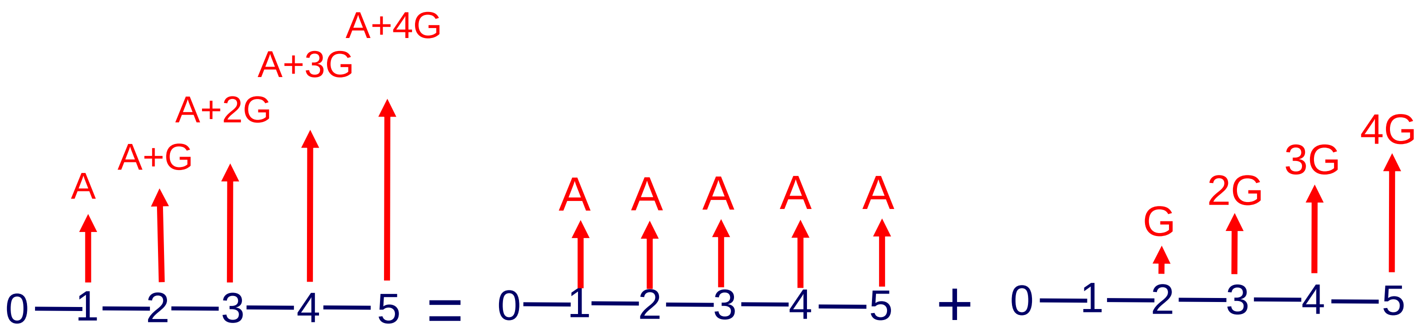
Uniform Series

$$(P/A, i, n) = \sum_{t=1}^{n} (P/F, i, t) \qquad (4\text{-}11)$$

$$(F/A, i, n) = 1 + \sum_{t=1}^{n-1} (F/P, i, t) \qquad (4\text{-}12)$$

$$(A/P, i, n) = (A/F, i, n) + i \qquad (4\text{-}13)$$

Arithmetic Gradient



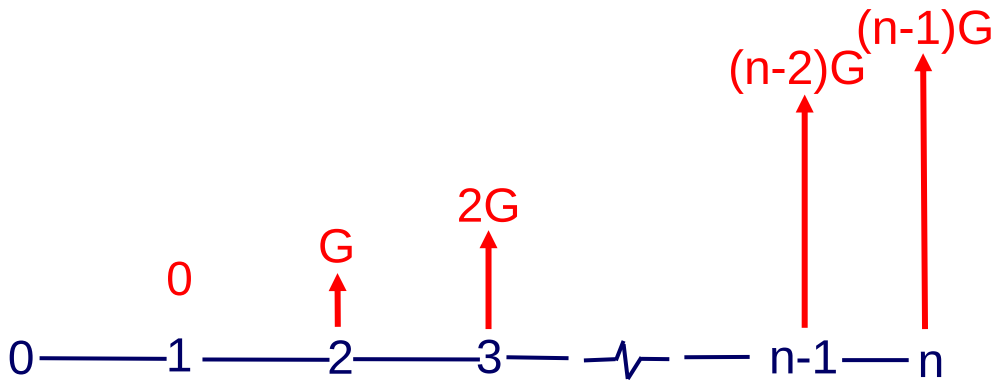
Examples:

- Operating & maintenance costs
- Salary packages

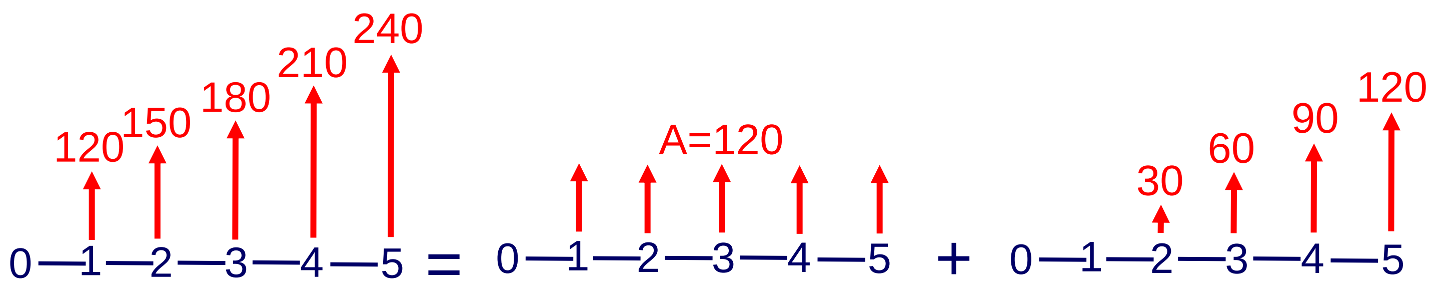
Arithmetic Gradient

Notation:

G = a fixed amount increment or decrement per time period



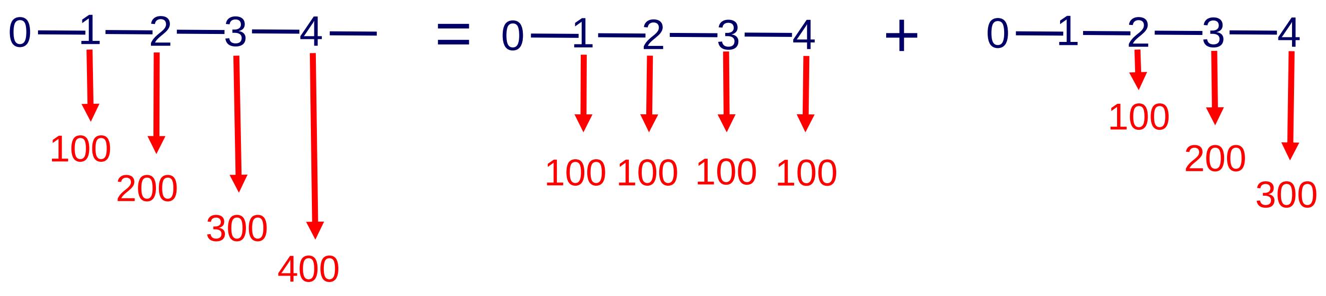
Example 4-11



Year	Cash Flow
1	120
2	150
3	180
4	210
5	240

$$
\begin{aligned}
P &= 120(P/A, 5\%, 5) + 30(P/G, 5\%, 5) \\
&= 120(4.329) + 30(8.237) \\
&= \$766
\end{aligned}
$$

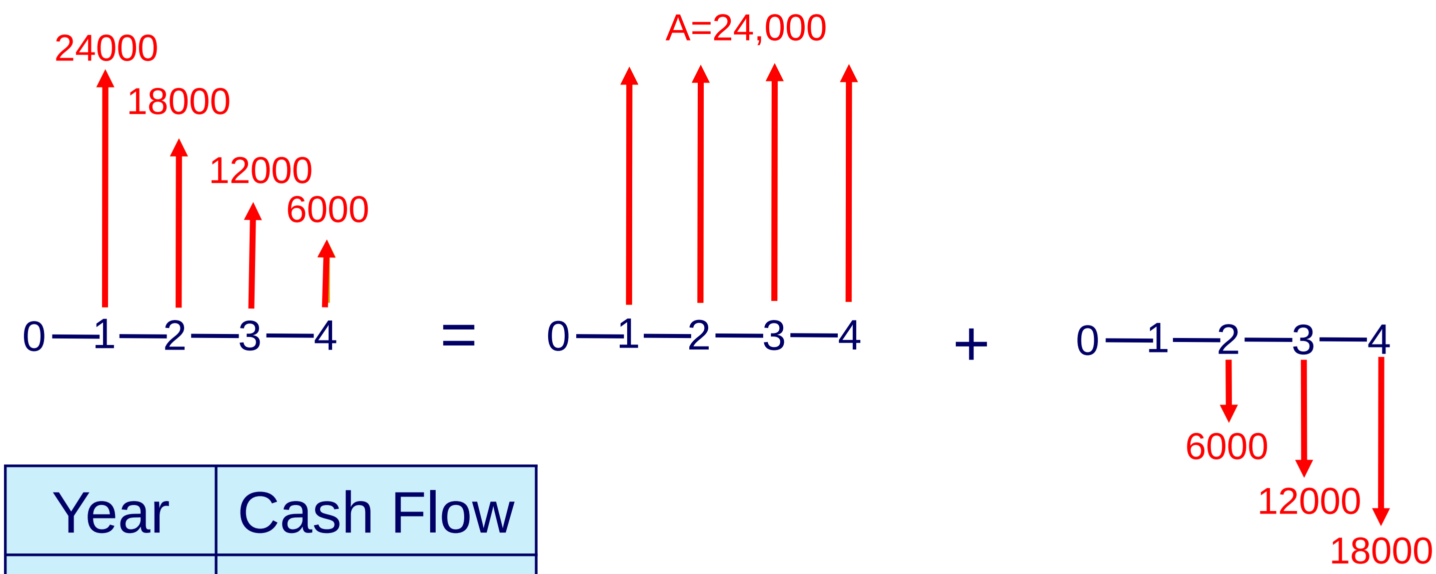
Example 4-12



Year	Cash Flow
1	100
2	200
3	300
4	400

$$A = 100 + 100(A/G, 6\%, 4)$$
$$= 100 + 100(1.427) = \$242.70$$

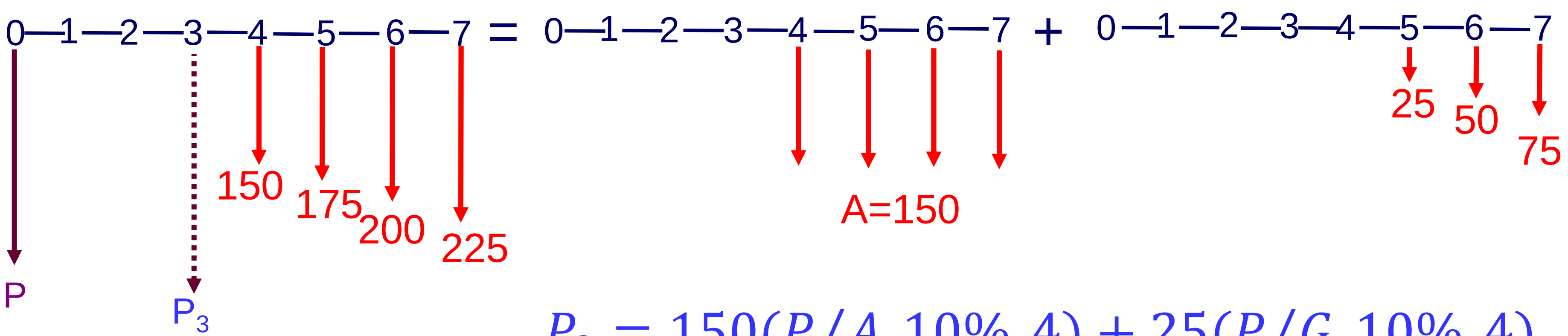
Example 4-13



Year	Cash Flow
1	24000
2	18000
3	12000
4	6000

$$A' = 24,000 - 6000(A/G, 10\%, 4)$$
$$= 24,000 - 6000(1.381) = \$15,714$$

Example 4-14



Year	Cash Flow
4	150
5	175
6	200
7	225

$$P_3 = 150(P/A, 10\%, 4) + 25(P/G, 10\%, 4)$$
$$= 150(3.170) + 25(4.378)$$
$$= \$584.95$$

$$P_0 = P_3(P/F, 10\%, 3)$$
$$= 584.95(0.7513)$$
$$= \$439.47$$

Reality & Assumed Uniformity of *A*, *G*, & *g*

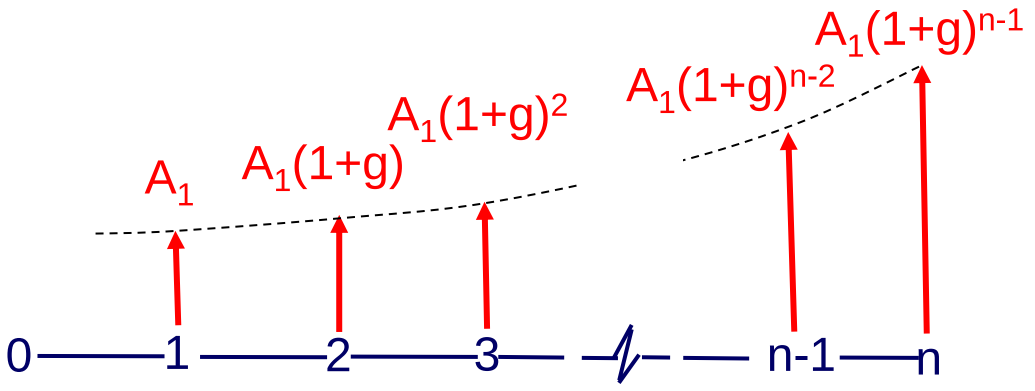
- Most future costs & benefits won't be uniform
  - Even so uniformity usually assumed
- Simpler models are easier to use
- Tabulated factors & spreadsheet annuity functions assume uniformity
- Engineering economy used in decision-making at feasibility & preliminary analysis stages
  - Not enough is known for estimates to be more detailed

Geometric Gradient

Notation:

g = a constant growth rate (+ or -) per period

A_1 = cash flow at period 1



Example 4-15 Geometric Gradient

At 8% interest find the PW of maintenance costs that are \$100 the first year & then increasing at 10% per year until the end year 5.

$$P = A_1\left[\frac{1-(1+g)^n(1+i)^{-n}}{i-g}\right]$$

$$= 100\left[\frac{1-(1+10\%)^5(1+8\%)^{-5}}{8\%-10\%}\right]$$

$$= \$480.42$$

Spreadsheets for Economic Analysis

1. Constructing tables of cash flows
2. Using annuity functions for *P*, *F*, *A*, *n*, or i
   - PV, FV, PMT, NPER, RATE
3. Block functions to find NPV or IRR
4. Making graphs
5. Conducting what-if analysis

Spreadsheet Annuity Functions (introduced in Chapter 3)

Excel Functions	**Purpose**
PV(RATE,NPER,PMT,[FV],[TYPE])	Find *P*
FV(RATE,NPER,PMT,[PV],[TYPE])	Find *F*
PMT(RATE,NPER,PV,[FV],[TYPE])	Find *A*
NPER(RATE,PMT,PV,[FV],[TYPE])	Find *n*
RATE(NPER,PMT,PV,[FV],[TYPE],[GUESS])	Find *i*

Build Amortization Table

- Borrow \$4000
- N = 5 years
- i = 10%
- Equal annual payments
- A =

Amortization Table

4000	Amount borrowed		
5	N		
10%	i		
$1,055.19	payment		
Period	Interest	Principal	Balance Due
0			4000.00
1	400.00	655.19	3344.81
2	334.48	720.71	2624.10
3	262.41	792.78	1831.32
4	183.13	872.06	959.26
5	95.93	959.26	0.00

Spreadsheet Block Functions

Excel Functions	Purpose
NPV (*i*, *CF_1:CF_n*)	To find net present value of a range of cash flows (from period 1 to n) at a given interest rate
IRR (*CF_0:CF_n*, [guess])	To find internal rate of return from a range of cash flows (from period 0 to n)

NPV & IRR are Block Functions for Cash Flow Tables

- Assume 1 cash flow per period
  - Equal length periods
  - Interest rate for that period
  - Not restricted to any pattern
- **0 must not be left as blank cell for cash flows**
- NPV (net present value) is a Present Worth
  - Periods 1 to *N* → **The first cell is *NOT* period 0 !**
- IRR (internal rate of return) is interest rate
  - PW at IRR = 0
  - Periods 0 to *N* → **The first cell *IS* period 0 !**
- Assumptions for period 0 are different, arbitrary, & critical.

Use the NPV Function

First calculate the NPV of the positive cash flows

=NPV(A1, B5:B9) = 216.47

Notice that this returns a positive number

PW = B4+NPV(A1,B5:B9)
= $16.47

	A	B
1	5%	interest rate
2		
3	Year	Cash Flow
4	0	-200
5	1	50
6	2	50
7	3	50
8	4	50
9	5	50
10	NPV, 1-5	$216.47
11	PW	$16.47

NPV, Different Cash Flows

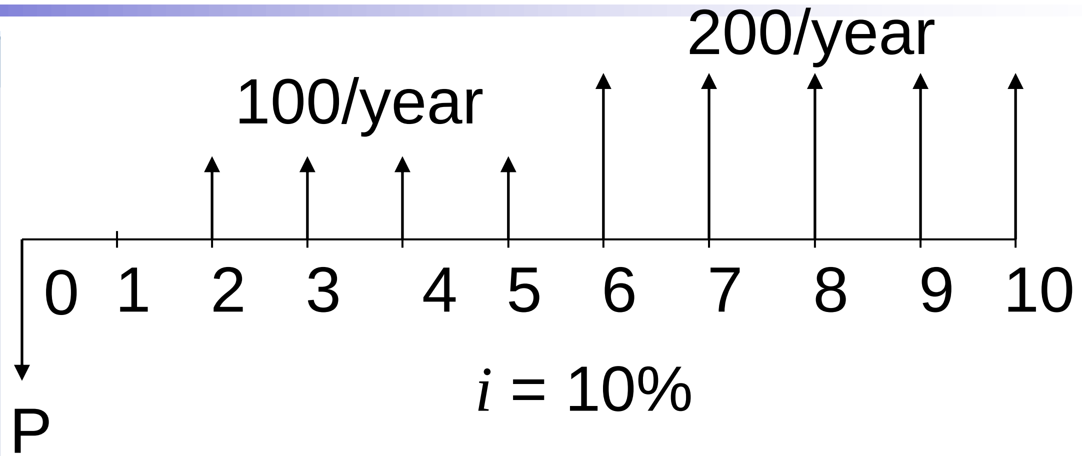
With different cash flows,
We cannot use the PV function.
Must use NPV function.

PW = B4 + NPV(A1, B5:B9)
= $12.25

	A	B
1	5%	interest rate
2		
3	Year	Cash Flow
4	0	-200
5	1	30
6	2	40
7	3	50
8	4	60
9	5	70
10	NPV, 1-5	$212.25
11	PW	$12.25

Another NPV Advantage

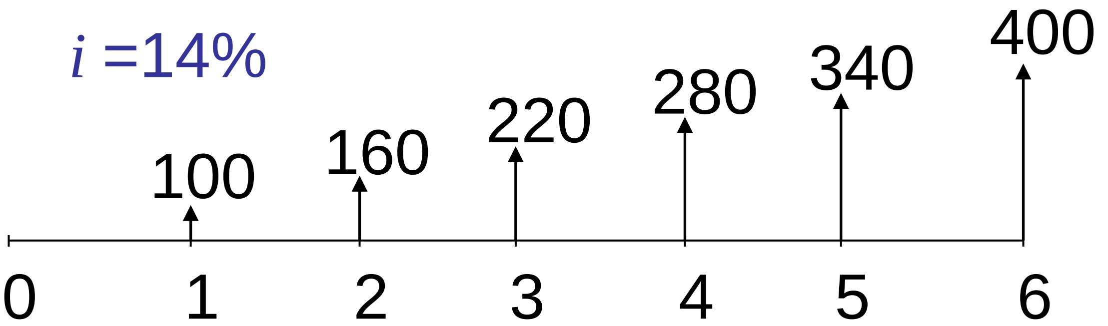
	A	B	C
1	Period	Cash Flow	
2	0		
3	1	0	
4	2	100	
5	3	100	
6	4	100	
7	5	100	
8	6	200	
9	7	200	
10	8	200	
11	9	200	
12	10	200	
13	NPV	$758.93	
14		=NPV(10%,B3:B12)	



Remember that NPV will discount the first cash flow, so start at year 1 & include it's zero value.

There is no G function in Excel
Use the NPV function

	A	B	C
1	Period	Cash Flow	
2	0		
3	1	100	
4	2	160	
5	3	220	
6	4	280	
7	5	340	
8	6	400	
9	NPV	$883.93	
10		=NPV(14%,B3:B8)	



IRR: =IRR(CF1:CF2)

What is the IRR?

=IRR(B4:B9) = 6.91%

At this rate the PW of the cash flows is 0.

	A	B
1	5%	interest rate
2		
3	Year	Cash Flow
4	0	-200
5	1	30
6	2	40
7	3	50
8	4	60
9	5	70
10	NPV, 1-5	$212.25
11	PW	$12.25
12	IRR	6.91%

Example 4-15 Geometric Gradient

EXAMPLE 4-15

The first-year maintenance cost for a new car is estimated to be $100, and it increases at a uniform rate of 10% per year. Using an 8% interest rate, calculate the present worth (PW) of the cost of the first 5 years of maintenance.

Example 4-15 Geometric Gradient

Spreadsheet approach

Note use of the data block; cells in Column B use cell referencing, are copied

	A	B	C
1	Example 4-15		
2	8%	interest rate	
3	5	years	
4	$100	initial cost	
5	10%	increase	
6			
7	Year	Cost	Formula
8	1	$100.00	=+A4
9	2	$110.00	=+B7*(1+A$5)
10	3	$121.00	=+B8*(1+A$5)
11	4	$133.10	=+B9*(1+A$5)
12	5	$146.41	=+B10*(1+A$5)
13	NPV	$480.43	=NPV(A2,B7:B11)

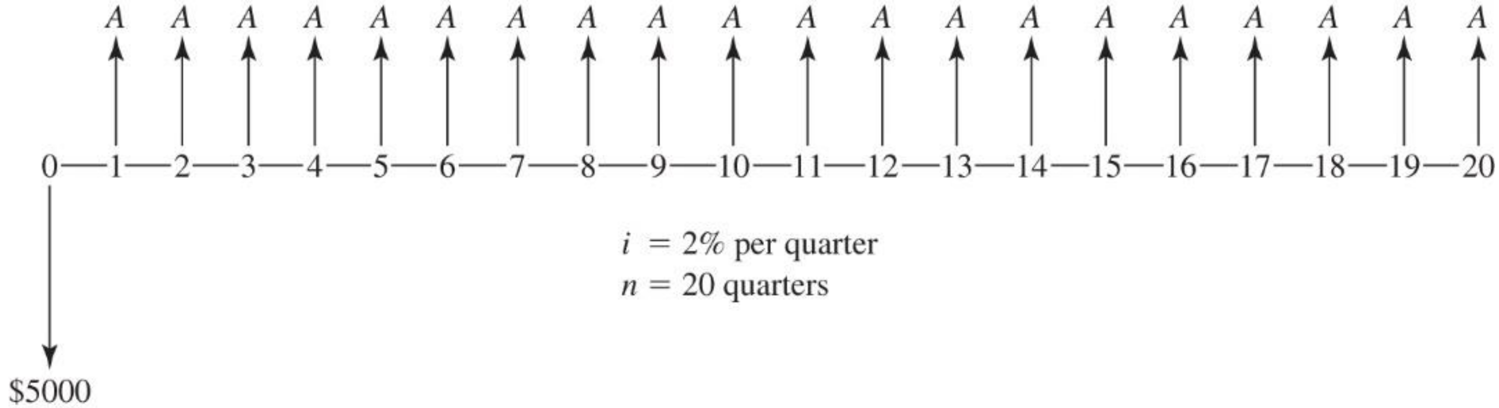
1st cost = $750,000. 1st year net revenue = $225,000, increasing either (a) $25,000 per yr. or (b) decreasing 10% per yr. or (c) increase by $25,000 for 1 yr. then decrease by 10% per yr. MARR = 12%; n = 5 yrs. Find PW & IRR for each scenario.

Example 4-16 Gradients

	A	B	C	D
1	$750,000	First cost		
2	12%	Interest rate		
3	$225,000	Year 1 net revenue		
4	Scenario	a	b	c
5	Gradient	Arithmetic	Geometric	
6		G	g	both
7	Value	$25,000	-10%	
8	Year			
9	0	-$750,000	-$750,000	-$750,000
10	1	225,000	225,000	225,000
11	2	250,000	202,500	250,000
12	3	275,000	182,250	225,000
13	4	300,000	164,025	202,500
14	5	325,000	147,623	182,250
15				
16	PW	$221,000	-$69,948	$42,448
17	Rate of Return	22.6%	7.9%	14.4%

Example 4-17 Compounding Period & Payment Period Differ

On Jan. 1, deposit $5000 that pays 8% nominal annual interest, compounded quarterly. Withdraw in 5 equal yearly sums, beginning December 31 of the first year. How much is withdrawn each year?



Compute equivalent A for each quarter

$A = P(A/P, i, n) = 5000(A/P, 2\%, 20) = 5000(0.0612) = \$306$

For each 1-year time period,

$W = A(F/A, i, n) = 306((F/A,2\%,4) = 306(4.122) = \$1260$